LESSON 2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

SECTION I

ELECTROSTATIC POTENTIAL

ELECTRIC FIELD IS CONSERVATIVE

In an electric field work done by the electric field in moving a unit positive charge from one point to the other, depends only on the position of those two points and does not depend on the path joining them.

ELECTROSTATIC POTENTIAL

Electrostatic potential is defined as **"Work required to be done against the force by electric field in bringing a unit positive charge from infinite distance to the given point in the electric field us called the electrostatic potential (V) at that point"**

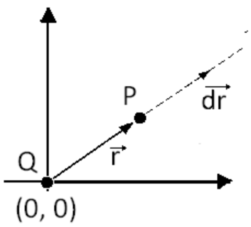
According to above definition the electric potential at point P is given by the formula

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr}$$

Electric potential is scalar quantity. SI units (J/C) called as volt (V)

POTENTIAL AT APOINT DUE TO A POINT CHARGE

Consider a point charge positive Q at the origin. To deter let P be the point at a distance 'r' from origin of coordinate axis . Since work done in electric field is independent of path, we will consider radial path as shown in figure.



According to definition of electric potential we can use the equation

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr}$$ And electric field E is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}$ given by

$$V_P = -\int_{\infty}^{P} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r} \cdot \vec{dr}$$

$$V_P = -\int_{\infty}^{P} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr$$

$$V_P = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]$$

$$V_P = \frac{Q}{4\pi\varepsilon_0 r}$$

ELECTRIC POTENTIAL DUE TO GROUP OF POINT CHARGE

The potential at any point due to group of point charges is the algebraic sum of the potentials contributed at the same point by all the individual point charges

$V = V_1 + V_2 + V_3 + ...$

ELECTRIC POTENTIAL DIFFERENCE

Electric potential difference is defined as "Work required to be done to take a unit positive charge from one point (say P) to another point (say Q) against the electric field

According to formula for potential at point P

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \overrightarrow{dr}$$

Thus potential at point Q is given by

$$V_Q = -\int_{\infty}^{Q} \vec{E} \cdot \overrightarrow{dr}$$

From above formula potential difference between points Q and P is given by

$$V_Q - V_P = \left(-\int_{\infty}^{Q} \vec{E} \cdot \overrightarrow{dr} \right) - \left(-\int_{\infty}^{P} \vec{E} \cdot \overrightarrow{dr} \right)$$

$$V_Q - V_P = \int_{Q}^{\infty} \vec{E} \cdot \overrightarrow{dr} + \int_{\infty}^{P} \vec{E} \cdot \overrightarrow{dr}$$

$$V_Q - V_P = \int_{Q}^{P} \vec{E} \cdot \overrightarrow{dr}$$

$$V_Q - V_P = -\int_{P}^{Q} \vec{E} \cdot \overrightarrow{dr}$$

Or work done in moving charge from point P to point Q

SI unit of potential is V and dimensional formula $[M^1L^2T^{-3}A^{-1}]$

ELECTROSTATIC POTENTIAL ENERGY

The electric potential energy is defined as "The work required to be done against the electric field in bringing a given charge *(q),* from infinite distance to the given point in the electric field motion without acceleration is called the electric potential energy of that charge at that point."

From definition of electric potential energy and the electric potential we can write electric potential energy of charge q at point P, as

$$U_p = -\int_{\infty}^{p} q\vec{E}.\overrightarrow{dr} = q\int_{\infty}^{p} \vec{E}.\overrightarrow{dr} = qV_p$$

The absolute value of the electric potential energy is not at all important, only the difference in its value is important. Here, in moving a charge *q,* from point P to Q, without acceleration, the work required to be done by the external force, shows the difference In the electric potential energies (U_Q - Up) of this charge *q,* at those two points.

$$U_Q - U_P = -q\int_P^Q \vec{E}.\overrightarrow{dr}$$

Electric potential energy is of the entire system of the sources producing the field and the charge, for some configuration, and when the configuration changes the electric potential energy of the system also changes.

POTENTIAL ENERGY OF A SYSTEM OF TWO POINT CHARGES

The potential energy possessed by a system of two – point charges q_1 and q_2 separated by a distance 'r' is the work done required to bring them to this arrangements from infinity. This electrostatic potential energy is given by

$$U = \frac{q_1q_2}{4\pi\varepsilon_0 r}$$

ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF POINT CHARGES

The electric potential energy of such a system is the work done in assembling this system starting from infinite separation between any two-point charges.
For a system of point charges q_1, q_2, q_3 …. q_n the potential energy is

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_iq_j}{4\pi\varepsilon_0 r_{ij}}\left(i \neq j\right)$$

It simply means that we have to consider all the pairs that are possible

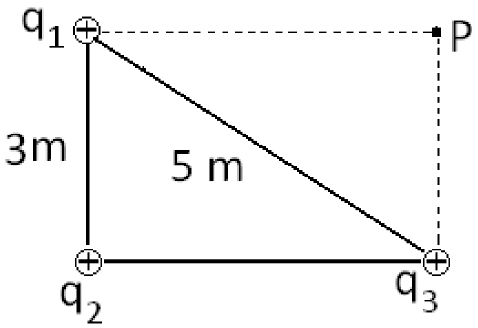
Important points regarding electrostatic potential energy
(i) Work done required by an external agency to move a charge *q* from *A* to B in an electric field with constant speed
(ii) When a charge *q* is let free in an electric field, it loses potential energy and gains kinetic energy, if it goes from *A* to B, then loss in potential energy = gain in kinetic energy
Or

$$q\left(V_B - V_A\right) = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2$$

Solved numerical

Q) Find work done by some external force in moving a charge q = 2μC from infinity to a point where electric potential is 10^4V
<u>Solution</u>
Work W = Vq = (10^4V)(2 $\times10^{-6}$C) = 2×10^{-2}J

Q) Three point charges $q_1 = 1\ \mu C$, $q_2 = -2\mu C$ and $q_3 = 3\mu C$ are fixed at position shown in figure (a) What is the potential at point P at the corner of the rectangle? (b) How much work would be needed to bring a charge $q_4 = 2.5\ \mu C$ from infinity to place it at P



Solution

(a) The total potential at point P is the scalar sum

$V_P = V_1 + V_2 + V_3$

$$V_P = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{1\times10^{-6}}{4} + \frac{-2\times10^{-6}}{5} + \frac{3\times10^{-6}}{3}\right)$$

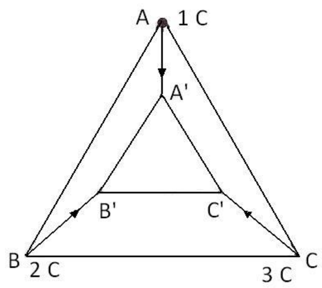
$$V_P = 9\times10^9\left(1.7\times10^{-6}\right)$$

$$V_P = 7.65\times10^3 V$$

(b) External work is $W_{ext} = q[V_f - V_i]$, In this case $V_i = 0$

So $W_{ext} = q_4 V_P = (2.5X10^{-6})(7.65X10^3) = 0.019J$

Q) Three point charges 1C, 2C, 3C are placed at the corner of an equilateral triangle of side 1m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5m as shown



Solution

As the potential energy of two point charges separated by a distance 'r' is given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

The initial potential energy of the system will be

$$U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times2}{1} + \frac{2\times3}{1} + \frac{3\times1}{1}\right]$$

$$U_i = 9\times10^9\times11$$

$$U_i = 9.9\times10^{10} J$$

The final potential energy of system

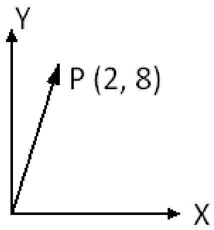
$$U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times 2}{0.5}+\frac{2\times 3}{0.5}+\frac{3\times 1}{0.5}\right]$$

$$U_f = 9\times 10^9 \times 22$$

$$U_f = 19.8\times 10^{10}\, J$$

So, the work done in changing the configuration of the system
W = U_f - U_i = (19.8 – 9.8)×10^{10} = 9.9×10^{10} J

Q) Suppose an electric field due to a stationary charge distribution is given by
$\vec{E} = ky\,\hat{\imath} + kx\,\hat{\jmath}$, where k is a constant. Obtain the formula for electric potential at any point on the line OP, with respect to (0, 0)



Solution
Equation of line is y = 4x
Let potential at origin is zero
In order to obtain potential at any point Q(x, y) on the line OP with respect to (0, 0) we can use

$$V_Q - V_P = -\int_{(0,0)}^{Q} \vec{E}\cdot\vec{dr}$$

$$V_Q - 0 = -\int_{(0,0)}^{Q} \left(ky\,\hat{i} + kx\,\hat{j}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$V_Q = -\int_{(0,0)}^{Q} \left(kydx + kxdy\right)$$

$$as\quad y = 4x$$

$$dy = 4dx$$

$$V_Q = -\int_{(0,0)}^{Q} 4kxdx + 4kxdx$$

$$V_Q = -\int_{0}^{x} 8kxdx$$

$$V_Q = -8k\left[\frac{x^2}{2}\right]_0^x$$

$$V_Q = 4kx^2$$

Q) The electric field at distance r perpendicularly from the length of an infinitely long wire is $E_{(r)} = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density of the wire. Find the potential at a point having distance b from the wire with respect to a point having distance a from the wire (a >b)

<u>Solution:</u> Let V_a be reference point thus $V_a = 0$

$$V_b - V_a = -\int_a^b \vec{E}\cdot\vec{dr}$$

$$V_b - V_a = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r} dr \therefore \left(\vec{E} \parallel \vec{dr}\right)$$

$$V_b - V_a = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r}$$

$$V_b - V_a = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln r\right]_a^b$$

$$V_b - V_a = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right)$$

$$\because V_a = 0 \ \ reference$$

$$V_{ba} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right)$$

Q) An electric field is represented by $\vec{E} = Ax\hat{\imath}$, where A = 10 V/m^2. Find the potential of the origin with respect to the point (10, 20) m

<u>Solution :</u> $\vec{E} = Ax\hat{\imath} = 10x\,\hat{\imath}$

$$V_{(0,0)} - V_{(10,20)} = -\int_{(10,20)}^{(0,0)} \vec{E}\cdot\vec{dr}$$

$$V_{(0,0)} - V_{(10,20)} = -\int_{(10,20)}^{(0,0)} \left(10x\,\hat{i}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$V_{(0,0)} - V_{(10,20)} = -\int_{10}^{0} \left(10x\,dx\right)$$

$$V_{(0,0)} - V_{(10,20)} = -10\left[\frac{x^2}{2}\right]_{10}^{0}$$

$$V_{(0,0)} - V_{(10,20)} = \left[0 - (-500)\right]$$

$$V_{(0,0)} - V_{(10,20)} = 500\ V$$

Since V(10,20) is to be taken zero V(0, 0) = 500 volts

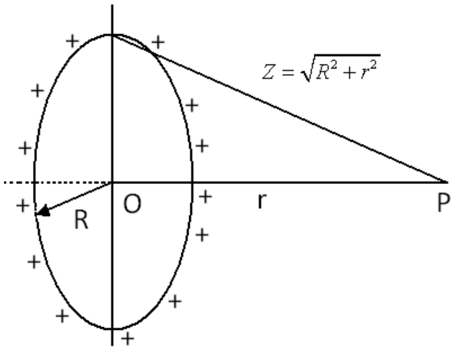
ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTION

The electric potential due to continuous charge distribution is the sum of potential of all the infinitesimal charge elements in which the distribution may be divided

V = $\int dV = \int \frac{dq}{4\pi\varepsilon_0 r}$

ELECTRIC POTENTIAL DUE TO A CHARGED RING

A charge Q is uniformly distributed over the circumference of a ring. Let us calculate the electric potential at an axial point at a distance r from the centre of the ring. The electric potential at P due to the charge element dq of the ring is given by



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2 + r^2}}$$

Hence electric potential at P due to the uniformly charged ring is given by

$$V = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2 + r^2}}\int dq$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2 + r^2}}$$

ELECTRIC POTENTIAL DUE TO A CHARGED DISC AT A POINT ON THE AXIS

A non-conducting disc of radius ‘R’ has a uniform surface charge density σ C/m^{2}to calculate the potential at a point on the axis of the disc at a distance from its centre. Consider a circular element of disc of radius x’ and thickness dx. All points on this ring are at the same distance $Z = \sqrt{x^2 + r^2}$, from the point P. The charge on the ring is dq = σA dq= σ(2πxdx) and so the potential due to the ring is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi x dx)}{\sqrt{x^2 + r^2}}$$

Since potential is scalar, there are no components. The potential due to the whole disc is given by

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{x}{\sqrt{x^2+r^2}}dx = \frac{\sigma}{2\varepsilon_0}\left[\left(x^2+r^2\right)^{1/2}\right]_0^R$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[\left(R^2+r^2\right)^{1/2} - r\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[r\left(\frac{R^2}{r^2}+1\right)^{1/2} - r\right]$$

For large distance R/r << 1 thus

$$r\left(\frac{R^2}{r^2}+1\right)^{1/2} \approx r\left(1+\frac{R^2}{2r^2}\right)$$

Substituting above value in equation for potential

$$V = \frac{\sigma}{2\varepsilon_0}\left[r\left(1+\frac{R^2}{2r^2}\right) - r\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\frac{R^2 r}{2r^2}$$

$$V = \frac{\sigma}{2\varepsilon_0}\frac{R^2}{2r}\left(\frac{\pi}{\pi}\right)$$

$$Since\ \ Q = \sigma\pi R^2$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Thus, at large distance, the potential due to disc is the same as that of point charge

ELECTRIC POTENTIAL DUE TO A SHELL

A shell of radius R has a charge Q uniformly distributed over its surface.

(a) At an external point

At point outside a uniform spherical distribution, the electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

Since $\vec{E}$ is radial, $\vec{E}.\overrightarrow{dr}$ = E dr

Since V(∞)=0, we have

$$V(r) = -\int_0^r \vec{E}\cdot\vec{dr} = -\int_0^r \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_\infty^r$$

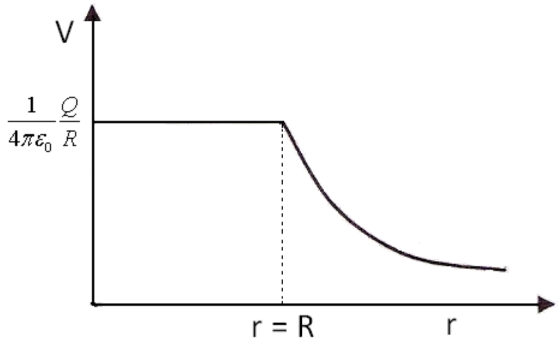
$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} \quad (r > R)$$

Thu potential due to uniformly charged shell is the same as that due to a point charge Q at the centre of the shell.

(b) At an internal point

At point inside the shell, E = 0. So work done in bringing a unit positive charge from a point on the surface to any point inside the shell is zero. Thus, the potential has a fixed value at all points within the spherical shell and is equal to the potential at the surface.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$



Above results hold for a conducting sphere also whose charge lies entirely on the outer surface.

ELECTRIC POTENTIAL DUE TO A NON-CONDUCTING CHARGED SPHERE

A charge Q is uniformly distributed through a non-conducting volume of radius R.

(a) Electric potential at external point is given by equation. ‘r’ is the distance of point from the center of the sphere

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(b) Electric potential at an internal point is given by equation

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R^3}\left[3R^2 - r^2\right]$$

Here R is the radius of the sphere and r is the distance of point from the centre

RELATION BETWEEN THE ELECTRIC FIELD AND ELECTRIC POTENTIAL

We know that electric potential from electric field is given by

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \overrightarrow{dr}$$

And potential difference between two points is given by

$$V_Q - V_P = -\int_{P}^{Q} \vec{E} \cdot \overrightarrow{dr}$$

If points P and Q are very close to each other, then for such a small displacement$\overrightarrow{dr}$, integration is not required and only term $\vec{E}.\overrightarrow{dr}$ can be kept thus
dV = - $\vec{E}.\overrightarrow{dr}$

I) If $\overrightarrow{dr}$, is the direction of electric field $\vec{E}$, $\vec{E}.\overrightarrow{dr} = Edrcos\theta = Edr$

$$dV = -Edr$$

$$E = -\frac{dV}{dr}$$

This equation gives the magnitude of electric field in the direction of displacement $\overrightarrow{dr}$.
Here $\boldsymbol{\frac{dV}{dr}}$ **= potential difference per unit distance. It is called the potential gradient.** It unit is Vm^{-1}, which is equivalent to N/C
II) If $\overrightarrow{dr}$, is not in the direction of $\vec{E}$, but in some other direction, the $-\boldsymbol{\frac{dV}{dr}}$ **would give us the component of electric field in the direction of that displacement**
If electric field is in X direction and displacement is in any direction (in three dimensions) then
$\vec{E} = E_X\hat{\imath}$ and $\overrightarrow{dr} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$
$\therefore$ dV = - $(E_X\hat{\imath}).(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k})$ = $-E_Xdx$
$\therefore E_x = -\frac{dV}{dx}$
Similarly, if the electric field is Y and only in Z direction respectively, we would get
$E_y = -\frac{dV}{dy}$ and $E_Z = -\frac{dV}{dz}$
Now if the electric field also have three (x,y,z) components then

$$E_x = -\frac{\partial V}{\partial x}. \quad E_y = -\frac{\partial V}{\partial y}. \quad E_Z = -\frac{\partial V}{\partial z}.$$

And $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right).$

Here $\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}$ shows the partial differentiation of V(x, y, z) with respect to x, y, z respectively. Moreover, the potential differentiation of V(x, y, z) with respect to x means the differentiation of V with respect to x only, by taking y and z in the formula of V as constant

Solved numerical

Q) The electric potential in a region is represented as V = 2x+3y-z. Obtain expression for the electric field strength

Solution

We know

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right).$$

Here

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[2\mathrm{x} + 3\mathrm{y} - \mathrm{z}] = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial x}[2\mathrm{x} + 3\mathrm{y} - \mathrm{z}] = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial x}[2\mathrm{x} + 3\mathrm{y} - \mathrm{z}] = -1$$

$$\vec{E} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$$

Q) The electrical potential due to a point charge is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ Find

a) the radial component of the electric field

b) the x-component of the electric field

Solution

a) The radial component of electric field

$E_r = -\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$.

(b) In terms of rectangular components, the radial distance r= $(x^2 + y^2+z^2)^{1/2}$; therefore the potential function

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2)^{\frac{1}{2}}}$$

To find the x-component of the electric field, we treat y and z constants. Thus

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2)^{\frac{3}{2}}}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{r^3}$$

EQUIPOTENTIAL SURFACE

An equipotential surface is that surface on which the electric potentials at all points are equal

Important points regarding equipotential surfaces

(i) The line of forces are always normal to equipotential surface.

(ii) The net work done in taking a charge from A to B is zero, if A and B are on same equipotential surface.

Suppose a unit positive charge is given a small displacement dl on the equipotential surface from a given point.

In this process the work required to be done against the electric field is

dW = $-\vec{E}.\overrightarrow{dl}$ = potential difference between those two point

But the potential difference on the equipotential surface = 0

$\therefore \vec{E}.\overrightarrow{dl} = 0 \Rightarrow$ E dl cosθ = 0, where θ = angle between $\vec{E}$ and $\overrightarrow{dl}$

But E ≠ 0 and dl ≠ 0

$\therefore \cos\theta = 0 \Rightarrow \theta = \pi/2$

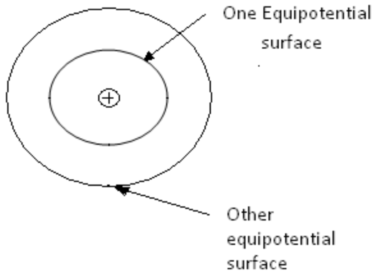
$\vec{E} \perp \overrightarrow{dl}$

But $\overrightarrow{dl}$ is along this surface. Hence electric field is normal to the equipotential surface at that point

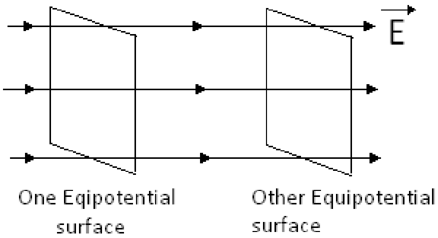
(iii) Equipotential surface never intersect each other. If they intersect then electric field lines will also intersect which is not possible.

Examples

(i) In the field of a point charge, the equipotential surfaces are spheres centred on the point charge.



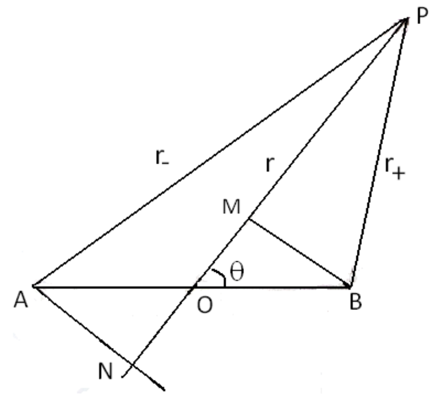
(ii) In a uniform electric field, the equipotential surfaces are planes which are perpendicular to the field lines.



(iii) In the field of an infinite line charge, the equipotential surfaces are co-axial cylinders having their axes at the line charge.

(iv)The surface of a conductor is an equipotential surface and the inside of conductor is equipotential space. Hence there is no electric field (and charge) inside the conductor's surface. The lines of forces are always normal to the surface of a conductor.

ELECTRIC POTENTIAL DUE TO DIPOLE



Let a dipole consisting of equal and opposite charge q separated by a distance 2a. Let zero of coordinate system is at centre of the dipole as shown in figure. Let p be any point in x-y plane. Let PO = r. AP = r_- BP = r_+ . Let r makes angle of θ with the axis of dipole.

Potential at point P is the sum of potential due at point P due to –q and +q charges.

BM is perpendicular on OP and AN is perpendicular on ON

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-}$$

$$V_{(P)} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

$$V_{(P)} = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_- - r_+}{r_- r_+}\right]$$

From figure r_- = r + ON but ON = acosθ

∴ r_- = r + acosθ

Similarly

r = r_+ + OM but OM = acosθ

∴ r = r_+ + a cosθ

r_+ = r – acosθ

r_- - r_+ = 2acosθ

Substituting the values we get

$$V_{(P)} = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{(r + a\cos\theta)(r - a\cos\theta)}\right]$$

$$V_{(P)} = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{(r^2 - a^2\cos^2\theta)}\right]$$

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2qa\cos\theta}{(r^2 - a^2\cos^2\theta)}\right]$$

$$V_{(P)} \approx \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Since r >>2a

Case I) Potential on the axis :

For point on the axis of the dipole θ 0 or π

$$V = \pm \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

From the given point, if the nearer charge is +q, then we get V as positive. And if it is –q, then we get V as negative

Case II) Potential on the equator

From a point on the equator $\theta = \pi/2 \therefore V = 0$

POTENTIAL ENERGY OF DIPOLE

When a dipole is placed in external uniform electric field $\vec{E}$, a torque $\vec{\tau} = \vec{p} \times \vec{E}$ acts on it. If we rotate the dipole through a small angle dθ, the work done by torque is

dW = τ dθ or dW = -PEsinθ dθ

The work is negative as the rotation dθ is opposite to the torque. The change in electric potential energy of the dipole is therefore

dU = -dW = -PEsinθ dθ

If dipole is rotated from angle θ_1 to θ_2, then

$$\int_{\theta_1}^{\theta_2} dU = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$U(\theta_2) - U(\theta_1) = pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$U(\theta_2) - U(\theta_1) = -pE\left(\cos\theta_2 - \cos\theta_1\right)$$

Work done by external force = $U(\theta_2) - U(\theta_1)$

OR $W_{ext} = pE(\cos\theta_1 - \cos\theta_2)$

Work done by electric force

$W_{electric\ force} = - W_{ext} = pE(\cos\theta_2 - \cos\theta_1)$

PERIODIC TIME OF DIPOLE

When a dipole is suspended in a uniform electric field, it will align itself parallel to the field. Now if it is given a small angular displacement θ about its equilibrium, the restoring couple will be

C = -pEsinθ

Or C = -pEθ [as for small θ, sinθ = θ]

Also couple

$$C = I\frac{d^2\theta}{dx^2}$$

Thus

$$I\frac{d^2\theta}{dx^2} = -pE\theta$$

$$\frac{d^2\theta}{dx^2} = -\frac{pE}{I}\theta$$

Comparing above equation with standard equation for SHM we get

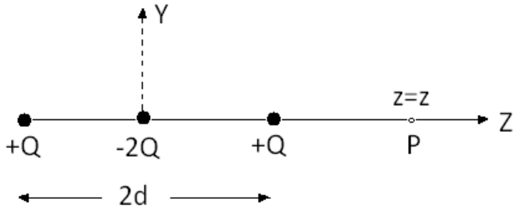
$$\omega^2 = \frac{pE}{I}$$

Thus periodic time

$$T = 2\pi\sqrt{\frac{I}{pE}}$$

Solved numerical

Q) When two dipoles are lined up in opposite direction, the arrangement is known as quadruple (as shown in figure) Calculate the electric potential at a point z = z along the axis of the quadruple



Electric potential at point p is given by

$V = V_1 + V_2 + V_3$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{z+d} + \frac{1}{z-d} - \frac{2}{z}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{2d^2}{z\left(z^2-d^2\right)}\right]$$

SECTION II

ELECTROSTATICS OF CONDUCTORS

a) EFFECT OF EXTERNAL FIELD ON CONDUCTOR

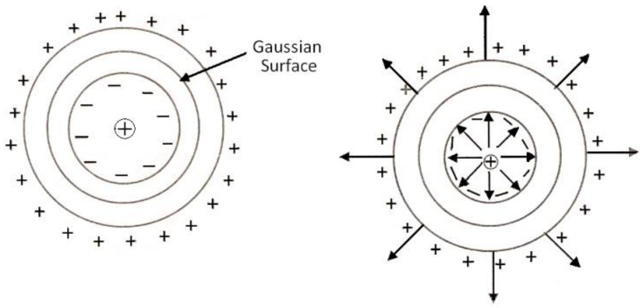
In a metallic conductor there are positive ions situated at the lattice points and the free electrons are moving randomly between these ions. They are free to move within the metal but not free to come out of the metal.

When such a conductor is placed in an external electric field, the free electrons move under the effect of the force in the direction opposite to the direction of electric field and get deposited on the surface of one end of the conductor, and equal amount of positive charge can be considered as deposited on the other end.

These induced charges produce an electric field inside the conductor, in the direction opposite to external electric field. When these two electric fields become equal in magnitude, the resultant net electric field inside the conductor becomes zero. Now the

motion of charges in the conductor stops and the charges become steady on the end surfaces
Thus in the case of metallic conductor, placed in an external electric field
(1) A steady electric charge distribution is induced on the surface of the conductor
(2) The net electric field inside the conductor is zero
(3) The net electric charge inside the conductor is zero
(4) On the outer surface of the conductor, the electric field at every point is locally normal to the surface.
If the electric field were not normal (perpendicular) a component of electric field parallel to the surface would exist and due to it the charge would move on the surface. But now the motion is stopped and the charges have become steady. Thus the component of electric field parallel to the surface would be zero, and hence the electric field would be normal to the surface.
(5) Since $\vec{E}$ =0 at every point inside the conductor, the electric potential everywhere inside the conductor and equal to the value of potential on the surface

CAVITY INSIDE A CONDUCTOR



Consider a +q_0 suspended in a cavity in a conductor. Consider. Consider a Gaussian surface just outside the cavity and inside the conductor $\vec{E}$ =0 on this Gaussian surface as it is inside the conductor form Gauss's law

$$\oint \vec{E}\,.\overrightarrow{ds} = \frac{\sum q}{\varepsilon_0}$$

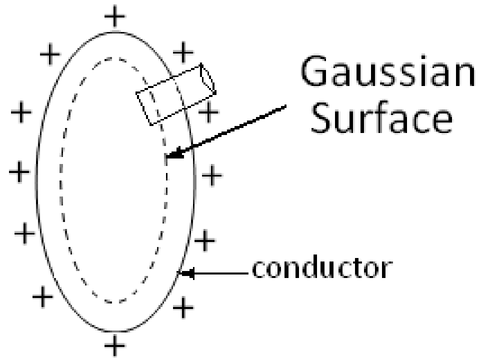
we have

$$\sum q = 0$$

This concludes that a charge of –q must reside on the metal surface on the cavity so that the sum of this induced charge –q and the original charge +q within the Gaussian surface is zero. In other words, a charge q suspended inside a cavity in a conductor is electrically neutral, a charge +q is induced on the outer surface of the conductor. As field inside the conductor, as shown in figure
ELECTROSTATIC SHIELDING
When a conductor with a cavity is placed inside the electric field, Charge induces on the surface of the conductor. These induce charges produce electric field inside the conductor such that net electric field inside the conductor and inside the cavity is zero. Thus electric field everywhere inside the cavity is zero. This fact is called electrostatic shielding

EFFECT PRODUCED BY PUTTING CHARGE ON THE CONDUCTOR

The charge placed on a conductor is always distributed only on the outer surface of the conductor. We can understand this by the fact that the electric field inside a conductor is zero. Consider a Gaussian Surface shown by the dots inside the surface and very close to it. Every point on it is inside the surface and not on the surface of conductor. Hence the electric field at every point on this surface is zero. Hence according to Gauss's theorem the charge enclosed by the surface is also zero



Consider a Gaussian surface of a pill-box of extremely small length and extremely small cross-section ds. A fraction of it is inside the surface and the remaining part is outside the surface . The total charge enclosed by this pill-box is q=σds, where σ = surface charge density of the charge on the conductor. At every point on the surface of the conductor $\vec{E}$ is perpendicular to the local surface element. Hence it is parallel to the surface vector $\overrightarrow{ds}$

But inside the surface $\vec{E}$ is zero. Hence flux coming out from the cross-section of pillbox inside the surface is zero. The flux coming out from the cross-section of pill-box outside the surface is $\vec{E}\,.\overrightarrow{ds} = E\ ds$

According to Gauss's theorem E ds = $\dfrac{q}{\varepsilon_0} = \dfrac{\sigma ds}{\varepsilon_0}$

$$E = \frac{\sigma}{\varepsilon_0}$$

In vector form

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

SECTION III

CAPACITORS AND CAPACITANCE

CAPACITY OF AN ISOLATED CONDUCTOR

When charge is given to an isolated body, its potential increases and the electric field also go on gradually increasing. In this process at some stage the electric field becomes sufficiently strong to ionize the air particles around the body as a result body is not able to store any additional charge. During the process the ratio of charge Q on the body and potential (V) on the body remains constant. This ratio is called the capacity of the body

$$C = Q/V$$

In SI system, the unit of capacity is coulomb/volt and is called Farad (F)

The capacity of a body is independent of the charge given to it and depends on the shape and size only.

CAPACITOR

Capacitor is an arrangement of two conductors carrying charges of equal magnitude and opposite sign and separated by an insulating medium. The following points may be carefully noted.

(i) The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that positively charged conductor has a charge +Q and the negatively charged conductor has a charge *-Q.*

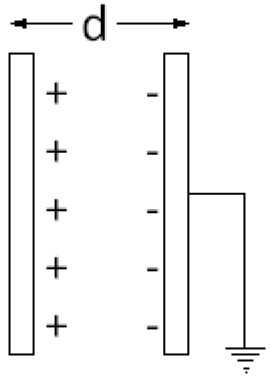
(ii) The positively charged conductor is at a higher potential than negatively charged conductor. The potential difference V between the conductors is proportional to the magnitude of charge Q and the ratio *Q/V* is known as capacitance C of the capacitor.

Q C= V

Unit of capacitance is farad (F). The capacitance is usually measured in microfarad (μF)

(iii) Circuit symbol is -||-

PARALLEL PLATE CAPACITOR



A parallel plate capacitor consists of two metal plates placed parallel to each other and separated by a distance *'d'* that is very small as compared to the dimensions of the plates. Due to this, the non-uniformity of the electric field near the ends of the plates can be neglected and in the entire region between the plates the electric field can be taken as constant. The area of each plate is *A.* Let Q be the charge on each plates. Surface charge is σ. Direction of electric field produced by both the plates is in same direction. Where outside the plates electric field is opposite in direction hence zero

Then electric field between the plates is given by

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The potential difference (V) between plates is given by V = Ed

$$V = \frac{\sigma}{\varepsilon_0} d = \frac{Q}{A\varepsilon_0} d$$

Hence

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

ISOLATED SPHERE AS A CAPACITOR

A conducting sphere of radius R carrying a charge Q can be treated as a capacitor with high potential conductor as the sphere itself and low potential conductor as sphere of infinite radius. The potential difference between these two spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 R} - 0$$

Hence Capacitance C = Q/V = $4\pi\varepsilon_0 R$

ENERGY STORED IN CHARGED CAPACITOR

In order to establish a charge on the capacitor, work has to be done on the charge. This work is stored in the form of the potential energy of the charge. Such a potential energy is called the energy of capacitor.

Suppose the charge on a parallel plate capacitor is Q. In this condition each plate of the capacitor is said to be lying in the electric field of the other plate.

The magnitude of the uniform electric field produced by one plate of capacitor is = $\frac{\sigma}{2\varepsilon_0}$

Where σ $is \frac{Q}{A}$ and A is area of plate

Hence taking arbitrarily the potential on this plate as zero, that of the other plate at distance d from it will be = $\frac{\sigma}{2\varepsilon_0}d$

The potential energy of the second plate will be = (potential) (charge Q on it)

Potential energy stored in capacitor =$\frac{\sigma}{2\varepsilon_0}dQ$

$$U_E = \frac{\sigma dQ}{2\varepsilon_0} = \frac{Q}{A}\frac{d\ Q}{2\varepsilon_0} = \frac{Q^2}{2\left({\varepsilon_0 A}/{d}\right)} = \frac{1}{2}\frac{Q^2}{C}$$

OR

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}VQ$$

ENERGY DENSITY OF A CHARGED CAPACITOR

Energy stored in capacitor is localized on the charges or the plates but is distributed in the field. Since in case of parallel plate capacitor, the electric field is only between the plates i.e. in a volume (A X d), the energy density

$$\rho_E = \frac{U_E}{volume} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

$$\rho_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

as E = V/d

$$\rho_E = \frac{1}{2}\varepsilon_0 E^2$$

FORCE BETWEEN THE PLATES OF A CAPACITOR

In a capacitor as plates carry equal and opposite charges, there is a force of attraction between the plates. To calculate this force, we use the fact that the electric field is conservative and in conservative field F = -dU/dx. In case of parallel plate capacitor

$$U_E = \frac{1}{2}\frac{Q^2}{C}$$

But $C = \frac{\varepsilon_0 A}{x}$

$$U_E = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A}x$$

So

$$F = -\frac{d}{dx}\left[\frac{1}{2}\frac{Q^2}{\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$$

The negative sign indicates that the force is attractive

Solved numerical

Q) The plates of a parallel plate capacitor are 5 mm apart and 2 m^2 in area. The plates are in vacuum. A potential difference of 1000 V is applied across a capacitor. Calculate

(a) the capacitance;

(b) the charge on each plate;

(c) the electric field in space between the plates;

(d) the energy stored in the capacitor.

Solution

(a) Capacitance

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times 10^{12}\times 2}{5\times 10^{-3}}$$

$$C = 0.0034\mu F$$

(b) Charge

Q = CV = (0.00354 X 10^{-6}) x (1000) = 3.54μC

The plate at higher potential has a positive charge +3.54 μC and the plate at lower potential has a negative charge of -3.54 μC

(c) E = V/d = 1000 / 0.005 = 2X10^5 V/m

(d) Energy = $\frac{1}{2}CV^2 = \frac{1}{2}(0.00354)\times 10^{-6}\times 10^6 = 1.77\times 10^{-3}J$

Q) A parallel plate air capacitor is made using two square plates each of side 0.2m spaced 1 cm apart. It is connected to a 50V battery

a) What is the capacitance?

b) What is the charge on each plate?

c) What is the electric field between the plates?

e) If the battery is disconnected and then the plates are pulled apart to a separation of 2cm, what are the answers to the above parts?

Solution

(a) $C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times 10^{-12})\times 0.2\times 0.2}{0.0.1} = 3.54\times 10^{-5}\mu F$

(b) Q = CV = 3.54x10^{-5}x50 = 1.77x10^{-3}μC

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}\times (3.54\times 10^{-11})(50^2) = 4.42\times 10^{-8}J$

(d) $E = V/d = 50/0.01 = 5000\ V/m$

(e) If the battery is disconnected the charge on the capacitor plates remains constant while the potential difference between the plates can change

$$C' = \frac{\varepsilon_0 A}{d} = \frac{C}{2} = 1.77\times10^{-5}\,\mu F$$

$$Q' = Q = 1.77\times10^{-3}\,\mu C$$

$$V' = \frac{Q'}{C'} = \frac{Q}{C/2} = 2V$$

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}\frac{C}{2}(2V)^2$$

$$U' = CV^2 = 8.84\times10^{-6}\,J$$

$$E' = \frac{V'}{d'} = \frac{2V}{2d} = E = 5000\,V/m$$

DIELECTRICS AND PLOLARISTAION

Non-conducting material are called dielectric. Dielectric materials are of two types (i) non-polar dielectric (ii) Polar dielectric

(i) Non-polar dielectric

In a non-polar molecule, **the centre of the positive and negative charge coincides with each other.** Hence they do not possess a permanent dipole moment. Now when it is placed in a uniform electric field, these centres are displaced in mutually opposite directions. Thus an electric dipole is induced in it or molecule is said to be polarized. If extent of electric field is not very strong, it is found that this dipole moment of molecule is proportional to external electric field $\vec{E}_0$

$$\therefore \vec{p} = \alpha\vec{E}_0$$

Where α is called the polarisability of the molecule

The units of α is $C^2\ m\ N^{-1}$

(ii) Polar Molecule

A polar molecule possesses a permanent dipole moment *p,* but such dipole moments of different molecules of the substance are randomly oriented in all possible directions and hence the resultant dipole moment of the substance becomes zero.

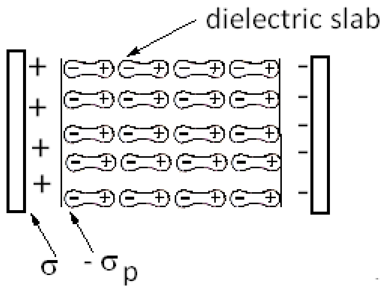
On applying an external electric field a torque acts on every molecular dipole. Therefore, it rotates and tries to become parallel to the electric field. Thus a resultant dipole moment is produced. In this way the dielectric made up of such molecules is said to be polarised.

Moreover, due to thermal oscillations the dipole moment also gets deviated from being parallel to electric field. If the temperature is T, the dipoles will be arranged in such an Equilibrium condition that the average thermal energy per molecule ($\frac{3}{2}k_BT$) balances the potential energy of dipole ($U = -\vec{p}.\vec{E}$) in the electric field. At 0 K temperature since the thermal energy is zero, the dipoles become parallel to the electric field.

EFFECT OF DIELECTRIC ON CAPACITANCE

Capacitance of a parallel plate capacitor in vacuum is given by charge density on plates is σ

$$C_0 = \frac{\varepsilon_0 A}{d}$$



Consider a dielectric inserted between the plates of capacitor, the dielectric is polarized by the electric field, the effect is equivalent to two charged sheets with surface charge densities σ_P and $-\sigma_P$. The electric field in the dielectric will be

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0}$$

So the potential difference across the plates is

$$V = Ed = \frac{\sigma - \sigma_P}{\varepsilon_0}\mathrm{d}$$

For linear dielectric, we expect σ_P to be proportional to electric field due to plates E_0
Thus $(\sigma - \sigma_P)$ is proportional to σ and can be written as
$(\sigma - \sigma_P) = \frac{\sigma}{K}$
Where K is a constant characteristic of the dielectric. Then we have

$$V = \frac{\sigma}{K\varepsilon_0}\mathrm{d} = \frac{\mathrm{Qd}}{\mathrm{A}K\varepsilon_0}$$

The capacitance C, with dielectric between the plates , is then

$$C = \frac{Q}{V} = \frac{K\varepsilon_0 A}{d} = \mathrm{KC_0}$$

The product $K\varepsilon_0$ is called the **permittivity** of the medium denoted by ε , $\varepsilon = K\varepsilon_0$
Or $K = \frac{\varepsilon}{\varepsilon 0}$
For vacuum K = 1 and for other dielectric medium K >1.

INTRODUCTION OF A DIELECTRIC SLAB OF DILECTRIC CONSTANT K BETWEEN THE PLATES

(a) When battery is disconnected

Let q_0, C_0, V_0, E_0 and U_0 represents the charge, capacity, potential difference, electric field and energy associated with charged air capacitor respectively. With the introduction of a dielectric slab of dielectric constant *K* between the plates and the battery disconnected.

(i) Charge remains constant, i.e., $q = q_0$, as in an isolated system charge is conserved.

(ii) Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes *K* times.

(iii) Potential difference between the plates decreases,

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}\ [\because q = q_0\ and\ C = KC_0]$$

$$V = \frac{V_0}{K}$$

(iv) As Field between the plates decreases,

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}\left[As\ V = \frac{V_0}{K}\right]$$

$$E = \frac{E_0}{K}$$

(v) Energy stored in the capacitor decreases

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}\quad [as\ q = q_0\ and\ C = KC_0]$$

(b) When battery remains connected (potential held constant)
(i) Potential remains constant i.e $V = V_0$
(ii) Capacity increases i.e $C = KC_0$
(iii) Charge on the capacitor increases i.e $q = Kq_0$
(iv) Electric field remains unchanged $E = E_0$
(v) Energy stored in the capacitor increases

Solved numerical

Q) A parallel plate capacitor has plates of area 4m^2 separated by a distance of 0.5 mm. The capacitor is connected across a cell of emf 100V
(a) Find the capacitance, charge and energy stored in the capacitor
(b) A dielectric slab of thickness 0.5mm is inserted inside this capacitor after it has been disconnected from the cell. Find the answers to part (a) if K = 3

<u>Solution:</u>
Part a

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85\times 10^{12}\times 4}{0.5\times 10^{-3}} = 7.08\times 10^{-2}\mu F$$

$$Q_0 = C_0V_0 = 7.08\times 10^{-2}\times 100 = 7.08\mu C$$

$$U_0 = \frac{1}{2}C_0V_0^2 = 3.54\times 10^{-4} J$$

Part b) As the cell has been disconnected $Q = Q_0$

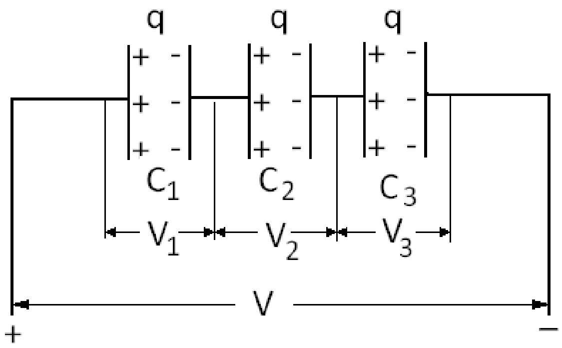
$$C = \frac{K\varepsilon_0 A}{d} = KC_0 = 0.2124\mu F$$

$$V = \frac{Q}{C} = \frac{Q}{KC_0} = \frac{V_0}{K} = \frac{100}{3}V$$

$$U_0 = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K} = 118\times10^{-6}J$$

GROUPING OF CAPACIORS

SERIES COMBINATION OF CAPACITORS



Capacitor are said to be connected in series if charge on each individual capacitor is same. In this situation

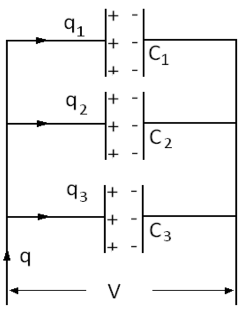
$V = V_1 + V_2 + V_3$

If C is the effective capacitance of combination then we know that $V = q/C$ and $V_1 = q/C_1$, $V_2 = q/C_2$, $V_3 = q/C_3$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

OR

PARALLEL COMBINATION OF CAPACITORS



When capacitors are connected in parallel, the potential difference V across each is same and the charge on C_1, C_2 and C_3 is different i.e q_1, q_2 and q_3

The total charge q is given by

$q = q_1 + q_2 + q_3$

potential across each capacitor is same thus $q_1 = C_1V$

$q_2 = C_2V$ and $q_3 = C_3V$

If C is equivalent capacitance then

$q = CV$

Thus $CV = C_1V + C_2V + C_3V$

Or $C = C_1 + C_2 + C_3$

Charge on capacitor

Let two capacitors are connected in parallel, let Q be the total charge , Let Q_1 be the charge on capacitor of capacity C_1 and Q_2 be the charge on capacitor of capacity C_2

Since both capacitor have same potential from formula $V = Q/C$ we get

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

$$\frac{Q_1+Q_2}{Q_2}=\frac{C_1+C_2}{C_2}$$

Since $Q = Q_1+Q_2$

$$\frac{Q}{Q_2}=\frac{C_1+C_2}{C_2}$$

$$Q_2=\frac{C_2}{C_1+C_2}Q$$

Similarly

$$Q_1=\frac{C_1}{C_1+C_2}Q$$

Solved numerical

Q) Two capacitors of capacitance C_1 = 6 μF and C_2 = 3 μF are connected in series across a cell of emf 18V. Calculate
(i) the equivalent capacitance
(ii) the potential difference across each capacitor
(iii) the charge on each capacitor

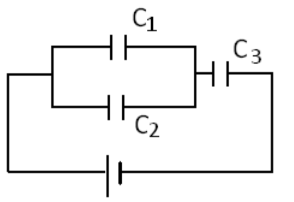
Solution

(i) $C=\frac{C_1C_2}{C_1+C_2}=\frac{6\times3}{6+3}=2\mu F$

(ii) $V_1=\left(\frac{C_2}{C_1+C_2}\right)V=\left(\frac{6}{6+3}\right)\times 18=12V$

(iii) In series combination charge on each capacitor is same Q = CV = $2\times10^{-6}\times18=36$ μC

Q) In the circuit shown the capacitors are C_1 = 15μF, C_2 = 10μF and C_3 = 25μF. Find



(i) the equivalent capacitance of the circuit
(ii) the charge on each capacitor
(iii) the potential difference across each capacitor

Solution

(i) C_1 and C_2 are parallel thus C_{12} = 15+10 = 25 μF

This C_{12} is in series with C_3 thus

$$\frac{1}{C}=\frac{1}{25}+\frac{1}{25}=\frac{2}{25}$$

C = 12.5 μF

(ii) Q is the total charge supplied by the cell = CV = (12.5 × 10) C

Charge on C_1 = Q_1 = $\left(\frac{C_1}{C_1+C_2}\right)Q=\left(\frac{15}{15+10}\right)\times125=75\mu C$

Charge on C_2 = Q_2 = $\left(\frac{C_2}{C_1+C_2}\right)Q=\left(\frac{10}{15+10}\right)\times125=50\mu C$

Charge on C_3 = Q = 125 μC

(iii) p.d across C_1 = V_1 = Q_1/C_1 = 75/15 = 5 V

p.d across $C_2 = V_2 = V_1 = 5$ V
p.d across $C_3 = V_3 = Q_3/C_3 = 125/25 = 5$V

REDISTRIBUTION OF CHARGES

If there are two spherical conductors of radius R_1 and R_2 at potential V_1 and V_2 respectively Far apart from each other (so that charge on one does not affect the other). The charges on them will be
$Q_1 = C_1V_1$ and $Q_2 = C_2V_2$
The total charge on the system is $Q = Q_1 + Q_2$
The capacitance $C = C_1 + C_2$
Now if they are connected through a wire, charge will flow from conductor at higher potential to lower potential till both acquires same potential let charge on first becomes q_1 and charge on second sphere becomes q_2
Since potential is same

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

We know that capacity of sphere $C = 4\pi\varepsilon_0 R$. Thus $C \propto R$

$$\frac{q_1}{R_1} = \frac{q_2}{R_2}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\frac{q_1 + q_2}{q_2} = \frac{R_1 + R_2}{R_2}$$

But $Q = q_1 + q_2$

$$\frac{Q}{q_2} = \frac{R_1 + R_2}{R_2}$$

$$q_2 = \frac{R_2}{R_1 + R_2} Q$$

similarly

$$q_1 = \frac{R_1}{R_1 + R_2} Q$$

Solved numerical

Q) Two isolated metallic solid spheres of radius R and 2R are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere

Solution: given both spheres have same charge density thus
$Q_1 = 4\pi R^2 \sigma$. And $Q_2 = 4\pi(2R)^2 \sigma$

Total charge on both spheres Q = 20 $\pi R^2 \sigma$

Now in sharing, charge is shared in proportion to capacity i.e. radius, so charge on the bigger spheres

$$q_2 = \frac{R_2}{R_1+R_2}Q = \frac{2R}{R+2R}Q = \frac{2Q}{3} \qquad \text{eq(1)}$$

So charge density on the bigger sphere after sharing

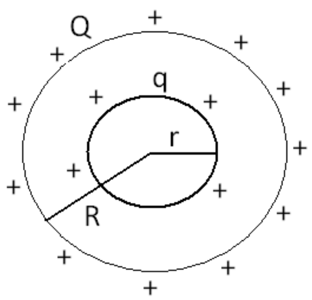
$$\sigma_2' = \frac{q_2}{4\pi(2R)^2} = \frac{2Q/3}{16\pi(R)^2} = \frac{Q}{24\pi(R)^2}$$

Putting the value of Q from equation (i) we get

$$\sigma_2' = \frac{20\pi R^2\sigma}{24\pi R^2} = \frac{5\sigma}{6}$$

VAN DE GRAFF GENERATOR

Principle:



Suppose there is a positive charge Q, on an insulated conducting spherical shell of radius R, as shown in figure. At the centre of this shell, there is a conducting sphere of radius r (r<R), having a charge q.

Here electric potential on the shell of radius R is,

$$V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{R}\right)$$

And the electric potential on the spherical shell of radius r is,

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{r}\right)$$

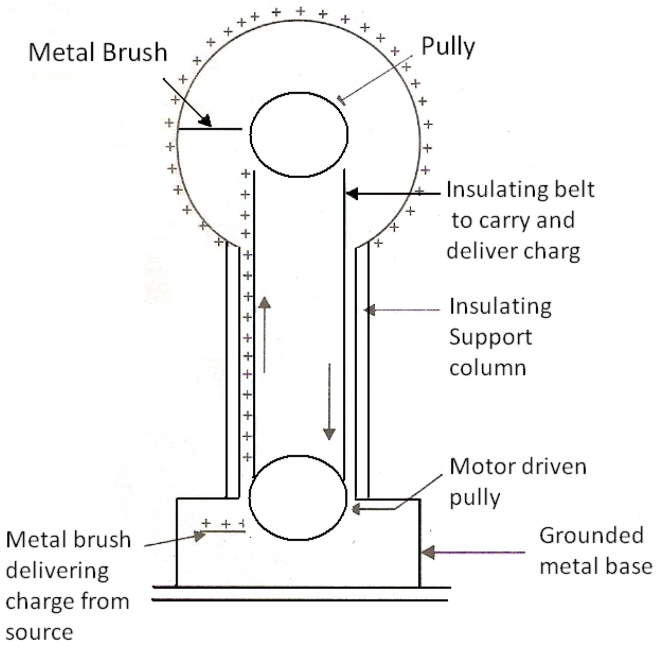
It is clear from these two equations that the potential on the smaller sphere is more and the potential difference between them is

$$V_r - V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{r}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{R}\right)$$

$$V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R}\right]$$

Hence if the smaller sphere is brought in electrical contact with bigger sphere then the charge goes from smaller to bigger sphere. Thus charge can be accumulated to a very large amount on the bigger sphere and there by its potential can be largely increased

Construction



As shown in the figure a spherical shell of a few meter radius, is kept on an insulated support, at a height of a few meters from the ground.
A pulley is kept at the centre of the big sphere and another pulley is kept on the ground. An arrangement is made such that a non-conducting belt moves across two pulleys. Positive charges are obtained from a discharge tube and are continuously sprayed on the belt using a metallic brush (with sharp edges) near the lower pulley. This positive charge goes with the belt towards the upper pulley.
There it is removed from the belt with the help of another brush and is deposited on the shell (because the potential on the shell is less than that of the belt on the pulley.) Thus a large potential difference (nearly 6 to 8 million volt) is obtained on the big spherical shell.
Uses : With the help of this machine, a potential difference of a few million (1 million = 10^6 = ten lac) volt can be established. By suitably passing a charged particle through such a high potential difference it is accelerated (to very high velocity) and hence acquires a very high energy ($\frac{1}{2}mv^2$). Because of such a high energy they are able to penetrate deeper into the matter. Therefore, fine structure of the matter can be studied with the help of them.

----------------END--------------

To become a member of www.gneet.com give call on 09737240300