NUCLEUS

Nucleus

The nucleus consists of the elementary particles, protons and neutrons which are known as nucleons. A proton has positive charge of the same magnitude as that of electron and its rest mass is about 1836 times the mass of an electron. A neutron is electrically neutral, whose mass is almost equal to the mass of the proton. The nucleons inside the nucleus are held together by strong attractive forces called nuclear forces.

A nucleus of an element is represented as $_ZX^A$,

Where, X = Chemical symbol of the element.

Z = Atomic number which is equal to the number of protons

A = Mass number which is equal to the total number of protons and neutrons.

The number of neutrons is represented as N which is equal to A−Z.

For example: The chlorine nucleus is represented as $_{17}Cl^{35}$. It contains 17 protons and 18 neutrons.

Atomic mass is expressed in atomic mass unit (u), defined as (1/12)th of the mass of the carbon (C^{12}) atom. According to this definition.

$$1u = \frac{1.992647 \times 10^{-26}}{12} kg$$

$$1u = 1.660539 \times 10^{-27} \text{ kg}$$

Classification of nuclei

(i) Isotopes

Isotopes are atoms of the same element having the same atomic number Z but different mass number A. The nuclei $_1H^1$, $_1H^2$ and $_1H^3$ are the isotopes of hydrogen. As the atoms of isotopes have identical electronic structure, they have identical chemical properties and placed in the same location in the periodic table.

The relative abundance of different isotopes differs from element to element. Chlorine, for example, has two isotopes having masses 34.98 u and 36.98 u, which are nearly integral multiples of the mass of a hydrogen atom. The relative abundances of these isotopes are 75.4 and 24.6 per cent, respectively. Thus, the average mass of a chlorine atom is obtained by the weighted average of the masses of the two isotopes, which works out to be

$$= \frac{(75.4 \times 34.98) + (24.6 \times 36.98)}{100} = 35.47\ u$$

(ii) Isobars

Isobars are atoms of different elements having the same mass number A, but different atomic number Z. The nuclei $_8O^{16}$ and $_7N^{16}$ represent two isobars. Since isobars are atoms of different elements, they have different physical and chemical properties.

(iii) Isotones

Isotones are atoms of different elements having the same number of neutrons. $_6C^{14}$ and $_8O^{16}$ are some examples of isotones.

(iv) Isomers

For some nuclei Z values are same and A values are also same but their radioactive properties are different. They are called isomers of each other. $_{35}B^{80}$ has one pair of isomers

Discovery of Neutron

James Chadwick who observed emission of neutral radiation when beryllium nuclei were bombarded with alpha-particles. (α-particles are helium nuclei).
It was found that this neutral radiation could knock out protons from light nuclei such as those of helium, carbon and nitrogen.
Application of the principles of conservation of energy and momentum showed that if the neutral radiation consisted of photons, the energy of photons would have to be much higher than is available from the bombardment of beryllium nuclei with α-particles.
The clue to this puzzle, which Chadwick satisfactorily solved, was to assume that the neutral radiation consists of a new type of neutral particles called *neutrons*.
From conservation of energy and momentum, he was able to determine the mass of new particle 'as very nearly the same as mass of proton'. Mass of neutron m_n = 1.00866 u
Or 1.6749×10^{-27} kg

General properties of nucleus

Nuclear size
According to Rutherford's α−particle scattering experiment, the distance of the closest approach of α − particle to the nucleus was taken as a measure of nuclear radius, which is approximately 10^{-15}m.
If the nucleus is assumed to be spherical, an empirical relation is found to hold good between the radius of the nucleus R and its mass number A. It is given by

$$R = R_0 A^{\frac{1}{3}}$$

Where, $R_0 = 1.2 \times 10^{-15}$ m. or is equal to 1.2 F (1 Fermi, F = 10^{-15} m)
This means the volume of the nucleus, which is proportional to R^3 is proportional to A.
Thus the density of nucleus is a constant, independent of *A.*

Nuclear density

The nuclear density ρ_N can be calculated from the mass and size of the nucleus

$$\rho_N = \frac{nuclear\ mass}{nuclear\ volume}$$

Nuclear mass = Am_N
where, A = mass number
m_N = mass of one nucleon and is approximately equal to 1.67×10^{-27} kg

Nuclear volume V_N

$$V_N = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3$$

$$\rho_N = \frac{Am_N}{\frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{m_N}{\frac{4}{3}\pi R_0^3}$$

Substituting the known values, the nuclear density is calculated as 1.816×10^{17} kg m^{-3} which is almost a constant for all the nuclei irrespective of its size. The high value of the nuclear density shows that the nuclear matter is in an extremely compressed state.

Nuclear mass

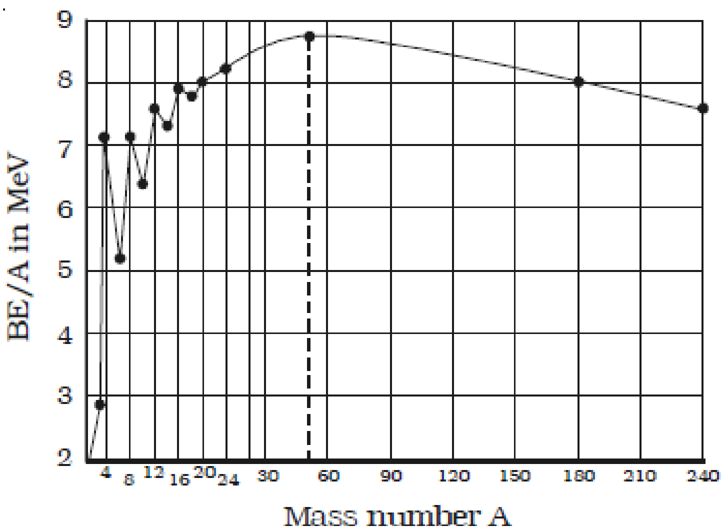
As the nucleus contains protons and neutrons, the mass of the nucleus is assumed to be the mass of its constituents.
Assumed nuclear mass = $Zm_P + Nm_N$,
Where, m_p and m_N are the mass of a proton and a neutron respectively
Z = number of protons
N = number of neutrons
However, from the measurement of mass by mass spectrometers, it is found that the mass of a stable nucleus (m) is less than the total mass of the nucleons.
i.e mass of a nucleus, $m < (Zm_p + Nm_N)$
$Zm_p + Nm_N - m = \Delta m$
where Δm is the mass defect
Thus, the difference in the total mass of the nucleons and the actual mass of the nucleus is known as the mass defect.

Note : In any mass spectrometer, it is possible to determine only the mass of the atom, which includes the mass of Z electrons.
If M represents the mass of the atom, then the mass defect can be written as

$$\Delta m = Zm_P + Nm_N + Zm_e - M$$

energy equivalent of 1 amu = 931 MeV

Binding energy



When the protons and neutrons combine to form a nucleus, the mass that disappears (mass defect, Δm) is converted into an equivalent amount of energy (Δmc^2). This energy is called the binding energy of the nucleus.
∴ Binding energy = $[Zm_P + Nm_N - m]c^2$
Binding energy = $\Delta m\, c^2$
The binding energy of a nucleus determines its stability against disintegration. In other words, if the binding energy is large, the nucleus is stable and vice versa.
The binding energy per nucleon is

$$\frac{BA}{A} = \frac{Binding\ energy\ of\ nucleus}{Total\ number\ of\ nucleons}$$

It is found that the binding energy per nucleon varies from element to element. A graph is plotted with the mass number A of the nucleus along the X−axis and binding energy per nucleon along the Y-axis.

Explanation of binding energy curve

(i) The binding energy per nucleon increases sharply with mass number A upto 20. It increases slowly after A = 20.
For A<20, there exists recurrence of peaks corresponding to those nuclei, whose mass numbers are multiples of four and they contain not only equal but also even number of protons and neutrons. Example: ${}_2He^4$, ${}_4Be^8$, ${}_6C^{12}$, ${}_8O^{16}$, and ${}_{10}Ne^{20}$.
The curve becomes almost flat for mass number between 30 and 170. Beyond 170, it decreases slowly as A increases.

(ii) The binding energy per nucleon reaches a maximum of 8.8 MeV at A=56, corresponding to the iron nucleus (${}_{26}Fe^{56}$). Hence, iron nucleus is the most stable.

(iii) The average binding energy per nucleon is about 8.5 MeV for nuclei having mass number ranging between 30 and 170. These elements are comparatively more stable and non radioactive.

(iv) For higher mass numbers the curve drops slowly and the BE/A is about 7.6 MeV for uranium. Hence, they are unstable and radioactive.

(v) The lesser amount of binding energy for lighter and heavier nuclei explains nuclear fusion and fission respectively. A large amount of energy will be liberated if lighter nuclei are fused to form heavier one (fusion) or if heavier nuclei are split into lighter ones (fission).

Nuclear force

The nucleus of an atom consists of positively charged protons and uncharged neutrons. According to Coulomb's law, protons must repel each other with a very large force, because they are close to each other and hence the nucleus must be broken into pieces. But this does not happen. It means that, there is some other force in the nucleus which overcomes the electrostatic repulsion between positively charged protons and binds the protons and neutrons inside the nucleus. This force is called nuclear force.

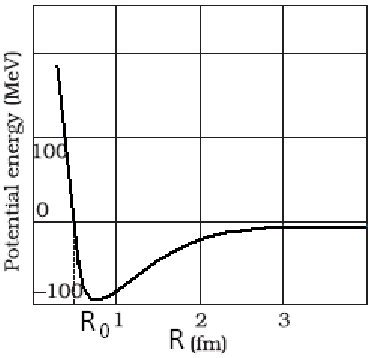
(i) Nuclear force is charge independent. It is the same for all the three types of pairs of nucleons (n−n), (p−p) and (n−p). This shows that nuclear force is not electrostatic in nature

(ii) Nuclear force is the strongest known force in nature. Nuclear force is about 1040 times stronger than the gravitational force.

(iii) Nuclear force is a short range force. It is very strong between two nucleons which are less than 10^{-15} m apart and is almost negligible at a distance greater than this. On the other hand electrostatic, magnetic and gravitational forces are long range forces that can be felt easily.

The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres. This leads to *saturation of forces* in a medium or a large-sized nucleus, which is the reason for the constancy of the binding energy per nucleon.

A rough plot of the potential energy between two nucleons as a function of distance is shown in the Fig.



The potential energy is a minimum at a distance R_0 of about 0.8 fm. This means that the force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.

However, the present view is that the nuclear force that binds the protons and neutrons is not a fundamental force of nature but it is secondary.

Radioactivity

The phenomenon of spontaneous emission of highly penetrating radiations such as α, β and γ rays by heavy elements having atomic number greater than 82 is called radioactivity and the substances which emit these radiations are called radioactive elements. The radioactive phenomenon is spontaneous and is unaffected by any external agent like temperature, pressure, electric and magnetic fields etc.
Experiments performed showed that radioactivity was a nuclear phenomenon in which an unstable nucleus undergoes a decay. This is referred to as *radioactive decay*. Three types of radioactive decay occur in nature :

(i) α-decay in which a helium nucleus $_2He^4$ is emitted.

(ii) β-decay in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;

(iii) γ-decay in which high energy (hundreds of keV or more) photons are emitted.
Each of these decay will be considered in subsequent sub-sections

Law of radioactive decay

In any radioactive sample, which undergoes α, β or γ-decay, it is found that the number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the sample. If N is the number of nuclei in the sample and ΔN undergo decay in time Δt then

$$\frac{\Delta N}{\Delta t} \propto N$$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where λ is called the radioactive *decay constant* or *disintegration constant*.
The change in the number of nuclei in the sample is dN = – ΔN in time Δt. Thus the rate of change of N is (in the limit Δt → 0)

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N \quad --- (1)$$

$$\frac{\mathrm{d}N}{N} = -\lambda \mathrm{d}t$$

Now, integrating both sides of the above equation, we get,

$$\int_{N_0}^{N} \frac{\mathrm{d}N}{N} = -\lambda \int_{t_0}^{t} dt$$

$$\ln N - \ln N_0 = -\lambda (t - t_0)$$

Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t0 and N is the number of radioactive nuclei at any subsequent time t. Setting $t_0 = 0$ and rearranging Equation gives us

$$ln \frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t} \quad --- (2)$$

Above equation represents law of radioactive decay
Differentiating equation (2) we get

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$$

Term -dN/dt is called the rate of disintegration or activity I of element at time t
From equation (1) we get I = λN
Thus

$$I = I_0 e^{-\lambda t}$$

Is alternative form of the law of *law of radioactive decay*
The SI unit for activity is becquerel, named after the discoverer of radioactivity, Henry Becquerel. It is defined as

1 becquerel = 1Bq = 1 decay per second

An older unit, the curie, is still in common use:

1 curie = 1 Ci = 3.7 × 10^{10} Bq (decays per second

Half life period

The half life period of a radioactive element is defined as the time taken for one half of the radioactive element to undergo disintegration.

From the law of disintegration

$$N = N_0 e^{-\lambda t}$$

Let $T_{½}$ be the half life period. Then, at t = $T_{½}$, N = $N_0/2$

$$\frac{N_0}{2} = N_0 e^{\lambda T_{1/2}}$$

$$\ln 2 = \lambda T_{½}$$

$$T_{1/2} = \frac{ln2}{\lambda}$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

Fraction of radioactive substance left undecayed is,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

where n is the number of half lives.

$$n = \frac{total\ time}{half\ life}$$

The half life period is inversely proportional to its decay constant. For a radioactive substance, at the end of T½, 50% of the material remain unchanged. After another T½ i.e., at the end of 2 T½, 25% remain unchanged. At the end of 3 T½, 12.5% remain unchanged and so on.

Solved Numerical

1. The half life of radon is 3.8 days. After how many days 19/20 of the sample will decay

Solution

If we take 20 parts as N_0 then N=1

From formula

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{1}{20} = \left(\frac{1}{2}\right)^n$$

$$20 = 2^n$$

using log we get log20=nlog2

1.3010 = n× 0.3010 thus n = 4.322

From formula

$$n = \frac{total\ time}{half\ life}$$

$$4.322 = \frac{t}{3.8}$$

t=16.42 days

Q) An archaeologist analysis of the wood in a prehistoric structure revels that the ratio of ^{14}C (half life = 5700 years) to ordinary carbon is only one fourth in the cells of living plants. What is the age of the wood?

Solution:

If we take N_0 = 1 then N = ¼

From formula

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{1}{4} = e^{-\lambda t}$$

$$4 = e^{\lambda t}$$

Taking log to the base e on both sides

ln4 = λt

Converting to log to base 10

2.303 log4 = λt

From formula for half life

$$\lambda = \frac{0.693}{T_{1/2}}$$

$$2.303\ log4\ = \frac{0.693}{T_{1/2}} t$$

$$t = \frac{2.303\ log4\ \times T_{1/2}}{0.693}$$

$$t = \frac{2.303\ \times 0.6021\ \times 5700}{0.693}$$

t =11400 years

Q) A radioactive nucleus X decays to nucleus Y with a decay constant $\lambda_X = 0.1\ s^{-1}$.

Y further decays to a stable nucleus Z with decay constant $\lambda_Y = \frac{1}{30} s^{-1}$

Initially there are only X nuclei and their number is $N_0 = 10^{20}$.

Set up the rate equation for the population of X, Y, and Z. The population of the Y nucleus as function of time is given by

$$N_Y = \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}\left(e^{-\lambda_y t} - e^{-\lambda_x t}\right)$$

Find the time at which N_Y is maximum and determine the population X and Z at that instant

Solution

Rate equation for X from Law of radioactive decay

$$\frac{dN_X}{dt} = -\lambda_X N_X \ \ ---eq(1)$$

Rate of decay of Y depends on generation of Y due to decay of X and population of Y at that instant thus

$$\frac{dN_Y}{dt} = \lambda_X N_X - \lambda_y N_y \ ---eq(2)$$

Rate of disintegration of Z depends only on rate of generation of Y thus

$$\frac{dN_Z}{dt} = \lambda_Y N_Y \ ---eq(3)$$

For N_Y to be maximum eq(2) should become zero

$$\lambda_X N_X - \lambda_y N_y = 0$$

$$\lambda_X N_X = \lambda_y N_y ---eq(4)$$

We know that

$$N_X = N_0 e^{-\lambda_x t} ---eq(5)$$

Given

$$N_Y = \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}\left(e^{-\lambda_y t} - e^{-\lambda_x t}\right) \ ---eq(6)$$

Substituting values of N_X and N_Y from equation (5) and (6) in equation (4) we get

$$\lambda_X N_0 e^{-\lambda_x t} = \lambda_y \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}\left(e^{-\lambda_y t} - e^{-\lambda_x t}\right)$$

$$e^{-\lambda_x t} = \lambda_y \frac{1}{\lambda_X - \lambda_Y}\left(e^{-\lambda_y t} - e^{-\lambda_x t}\right)$$

$$\frac{\lambda_X - \lambda_Y}{\lambda_y} = \frac{\left(e^{-\lambda_y t} - e^{-\lambda_x t}\right)}{e^{-\lambda_x t}}$$

$$\frac{\lambda_X}{\lambda_y} - 1 = \frac{e^{-\lambda_y t}}{e^{-\lambda_x t}} - 1$$

$$\frac{\lambda_X}{\lambda_y} = e^{(\lambda_x t - \lambda_y t)}$$

Taking log on both side

$$(\lambda_x - \lambda_y)t = ln\left(\frac{\lambda_X}{\lambda_y}\right)$$

$$t = \frac{1}{(\lambda_x - \lambda_y)} ln\left(\frac{\lambda_X}{\lambda_y}\right)$$

$$t = \frac{1}{0.1 - \frac{1}{30}} ln\left(\frac{0.1}{1/30}\right)$$

$$t = 15ln(3)$$
$$t = 2.303 \times 15 \times log(3)$$

t=16.48s is time when population of Y is maximum

To find population of X and Z at t = 16.48s

We will use equation

$$N_X = N_0 e^{-\lambda_x t}$$

$$N_X = 10^{20} \times e^{-0.1\times16.48} = 10^{20}\frac{1}{e^{1.648}}$$

[Calculation of $e^{1.648}$

$\log_{10}(e^{1.648})$ =(1.648)$\log_{10}e$

$\log_{10}(e^{-1.68})$ =(1.648) × 0.434 = 0.7155

antilog (0.7155) = 5.194

thus value of $e^{1.648}$ = 5.194]

$\therefore N_X = 10^{20} \times \frac{1}{5.194} = 1.925 \times 10^{19}$

From equation (4)

$$\lambda_X N_X = \lambda_y N_y$$
$$N_Y = N_X \frac{\lambda_X}{\lambda_y}$$
$$N_Y = 1.925 \times 10^{19} \times \frac{0.1}{1/30} = 3 \times 1.925 \times 10^{19} = 5.772 \times 10^{19}$$

Now $N_Z = N_0 - N_X - N_Y$

$N_z = (10\times10^{19}) - (1.925\times10^{19}) - (5.772\times10^{19}) = 2.303\times10^{19}$

Q) In a mixture of two elements A and B having decay constants 0.1 day^{-1} and 0.2 day^{-1} respectively; initially the activity of A is 3 times that of B. If the initial activity of the mixture is 2mCi, find the activity of it after 10 days

Solution:

λ_A= 0.1 day^{-1} λ_B=0.2 day^{-1}

$(I_0)_A = 3(I_0)_B$

At time t = 0, activity of mixture is

$I_0 = (I_0)_A + (I_0)_B$

$I_0 = 3(I_0)_B + (I_0)_B$

$2 = 4(I_0)_B$
$(I_0)_B = 0.5$ mCi
$(I_0)_A = 1.5$ mCi
At time t , activity of A is

$$I_A = (I_0)_A e^{-\lambda_A t} = (1.5)e^{-(0.1)(10)}$$
$$I_A = \frac{1.5}{e} = \frac{1.5}{2.718} = 0.552\ mCi$$

At time t , activity of B is

$$I_B = (I_0)_B e^{-\lambda_B t} = (0.5)e^{-(0.2)(10)}$$
$$I_B = \frac{1.5}{e^2} = \frac{1.5}{(2.718)^2} = 0.067\ mCi$$

At time t, total activity of the mixture
$I = I_A + I_B = 0.552 + 0.067 = 0.619$ mCi

Mean life (τ)

The time-interval, during which the number of nuclei of a radioactive element becomes equal to the e^{th} part of its original number, is called the mean life or average life τ of the element .
When $N = N_0/e$, we can put t = mean life = τ

$$\therefore \frac{N_0}{e} = N_0 e^{-\lambda\tau}$$

$$e = e^{\lambda\tau}$$

$$\tau = \frac{1}{\lambda}$$

Thus mean life is equal to the reciprocal of the decay constant
Relation between $T_{1/2}$ and mean life

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$T_{1/2} = 0.693\tau$$

Solved Numerical

Q) A radioactive sample emits n β particles in 2 second. In next 2 seconds, it emits 0.75n β particles. What is the mean life of the sample

Solution:

Disintegration of one nucleon give one β particle
If n β-particles are emitted then N-n nucleons are not disintegrated thus
$N-n = Ne^{-\lambda 2}$
$n = N(1-e^{-2\lambda})$ ----eq(1)

In 4 seconds total emission o is n +0.75n = (1.75)n thus
(1.75)n =N(1-$e^{-4\lambda}$) ---eq(2)
Dividing eq(2) by eq(1)

$$1.75 = \frac{1-e^{-4\lambda}}{1-e^{-2\lambda}}$$
$$\frac{7}{4} = \frac{1-e^{-4\lambda}}{1-e^{-2\lambda}}$$
$$7 - 7e^{-2\lambda} = 4 - 4e^{-4\lambda}$$
$$4e^{-4\lambda} - 7e^{-2\lambda} + 3 = 0$$
$$4e^{-4\lambda} - 4e^{-2\lambda} - 3e^{-2\lambda} + 3 = 0$$
$$4e^{-2\lambda}(e^{-2\lambda} - 1) - 3(e^{-2\lambda} - 1) = 0$$
$$(\mathrm{e}^{-2\lambda} - 1)(4e^{-2\lambda} - 3) = 0$$
$$\text{But } \mathrm{e}^{-2\lambda} - 1 \neq 0$$
$$\therefore 4e^{-2\lambda} - 3 = 0$$
$$e^{-2\lambda} = \frac{3}{4}$$
$$e^{2\lambda} = \frac{4}{3}$$
$$2\lambda = ln\left(\frac{4}{3}\right)$$
$$\frac{1}{\lambda} = \frac{2}{ln\left(\frac{4}{3}\right)} = \frac{2}{ln4 - ln3}$$
$$\tau = \frac{2}{ln4 - ln3}$$

Radioactive displacement law

During a radioactive disintegration, the nucleus which undergoes disintegration is called a parent nucleus and that which remains after the disintegration is called a daughter nucleus. In 1913,Soddy and Fajan framed the displacement laws governing radioactivity.

α-decay

When a radioactive nucleus disintegrates by emitting an α-particle, the atomic number decreases by two and mass number decreases by four. The α-decay can be expressed as
$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$
example, when $_{92}U^{238}$ undergoes alpha-decay, it transforms to $_{90}Th^{234}$

$$\mathbf{{}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}}$$

The alpha-decay of $_{92}U^{238}$ can occur spontaneously (without an external source of energy) because the total mass of the decay products $_{90}Th^{234}$ and 2He4 is less than the mass of the original $_{92}U^{238}$.

Thus, the total mass energy of the decay products is less than the mass energy of the original nuclide.
The difference between the initial mass energy and the final mass energy of the decay products is called the *Q value* of the process or the *disintegration energy*.
Thus, the *Q* value of an alpha decay can be expressed as

$$Q = (m_X - m_Y - m_{He})\, c^2$$

This energy is shared by the daughter nucleus and the alpha-particle, in the form of kinetic energy. Alpha-decay obeys the radioactive law

β–*decay*

In the process of β-decay, a nucleus spontaneously emits electron or positron. Positron has the same charge as that of electron but it is positive and its other properties are exactly identical to those of electron. Thus positron and the antiparticle of electron. Positron and electron are respectively written as β^+ and β^- or ${}_{+1}e^0$ and ${}_{-1}e^0$ and e^+ and e^-

β^- emission

$${}_{15}P^{32} \rightarrow {}_{16}S^{32} + {}_{-1}e^0 + \bar{\upsilon} \quad (\bar{\upsilon} \text{ is called antineutrino})$$

Compared to parent element, the atomic number of daughter element is one unit more in β^- decay
In this reaction neutron disintegrates into proton can be sated as

$$N^0 \rightarrow P^{+1} + \beta^- + \bar{\upsilon}$$

β^+ emission

$${}_{11}Na^{22} \rightarrow {}_{10}Ne^{23} + {}_{+1}e^0 + \upsilon \quad (\upsilon \text{ is called neutrino})$$

Compared to parent element, the atomic number of daughter element is one unit less in β^+ decay
In this reaction Proton disintegrates into Neutron can be sated as

$$P^{+1} \rightarrow N^0 + \beta^+ + \upsilon$$

Neutrino and anti-neutrino are the anti particles of each other. They are electrically neutral and their mass is extremely small as compared to even that of electron. Their interaction with other particles is negligible and hence it is extremely difficult to detect them. They can pass without interaction even through very large matter (even through the entire earth). They have h/2π spin

γ–decay

There are energy levels in a nucleus, just like there are energy levels in atoms. When a nucleus is in an excited state, it can make a transition to a lower energy state by the emission of electromagnetic radiation.
As the energy differences between levels in a nucleus are of the order of MeV, the photons emitted by the nuclei have MeV energies and are called **gamma rays**.
Most radio-nuclides after an alpha decay or a beta decay leave the daughter nucleus in an excited state. The daughter nucleus reaches the ground state by a single transition or sometimes by successive transitions by emitting one or more gamma rays.
A well-known example of such a process is that of ${}_{27}Co^{60}$.
By beta emission, the ${}_{27}Co^{60}$ nucleus transforms into ${}_{28}Ni^{60}$ nucleus in its excited state.
The excited ${}_{28}Ni^{60}$ nucleus so formed then de-excites to its ground state by successive emission of 1.17 MeV and 1.33 MeV gamma rays.

Nuclear reactions

By bombarding suitable particles of suitable energy on a stable element, that element can be transformed into another element. Such a reaction is called artificial nuclear transmutation. Example

$$\mathbf{{}_{7}N^{14} + {}_{2}He^{4} \rightarrow {}_{8}O^{17} + {}_{1}H^{1} + Q}$$

Such process, in which change in the nucleus takes place are called nuclear reactions. Here Q is called Q –value of the nuclear reaction and it shows that the energy released in the process. If Q>0, the reaction is exoergic and if Q<0 then reaction is endoergic
Reaction can be symbolically represented as

$$\mathbf{A + a \rightarrow B + b + Q}$$

A : is called the target nucleus
a: is called projectile partile
B: is called product nucleus
b: is called emitted particle

The energy liberated $Q = [m_A + m_a – m_B – m_b]\, c^2$, here m represents the mass of respective particle

Nuclear Fission

The process of breaking up of the nucleus of a heavier atom into two fragments with the release of large amount of energy is called nuclear fission.

The fission is accompanied of the release of neutrons.
The fission reactions with $_{92}U^{235}$ are represented as

$$_{92}U^{235} + {_0}n^1 \rightarrow {_{56}}Ba^{141} + {_{36}}Kr^{92} + 3\,{_0}n^1 + Q \;...(1)$$
$$_{92}U^{235} + {_0}n^1 \rightarrow {_{54}}Xe^{140} + {_{38}}Sr^{94} + 2\,{_0}n^1 + Q \;...(2)$$
$$_{92}U^{235} + {_0}n^1 \rightarrow {_{51}}Sb^{140} + {_{41}}Nb^{99} + 4\,{_0}n^1 + Q \;...(3)$$

The product nuclei obtained by the fission are called the fission fragments, the neutrons are called the fission neutrons and energy is called fission energy. In the above reaction 60 different nuclei are obtained as fission fragment, having Z value between 36 and 56. The probability is maximum for formation of nucli with A = 95 and A = 140. The fission fragments are radio active and by successive emission of β^- particles results in stable nucli. The disintegration energy in fission events first appears as the kinetic energy of the fragments and neutrons. Eventually it is transferred to the surrounding matter appearing as heat.

Solved Numerical

Q In the reaction $_ZX^A \rightarrow {_{z-2}}Y^{A-4} + {_2}He^4$ + Q of the nucleus X at rest, taking the ratio of mass of α-particle M_α and mass of Y-nucleus as

$$\frac{M_\alpha}{M_\beta} = \frac{4}{A-4}$$

Show that the Q-value of the reaction is given by

$$Q = K_\alpha\left(\frac{A}{A-4}\right)$$

K_α = kinetic energy of α particles

Solution:

Q-value of reaction = energy equivalent to mass-difference

$$Q = (M_X - M_Y - M_\alpha)c^2$$

Q = increase in kinetic energy

Q = ($K_\alpha + K_\beta$) – 0 (∴ X was steady)

$$Q = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_\beta v_\beta^2 \quad ----eq(1)$$

From conservation of momentum

$$M_Y\vec{v}_Y + M_Y\vec{v}_Y = 0$$

$M_Y v_Y = M_\alpha v_\alpha$ (in magnitude)

$$v_Y = \left(\frac{M_\alpha}{M_Y}\right)v_\alpha$$

Substituting this value in equation (1)

$$Q = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y\left(\frac{M_\alpha}{M_Y}\right)^2 {v_\alpha}^2$$

$$Q = \frac{1}{2}M_\alpha v_\alpha^2\left[\frac{M_\alpha}{M_Y} + 1\right]$$

$$Q = K_\alpha \left[\frac{4}{A-4} + 1\right]$$
$$Q = K_\alpha \left(\frac{A}{A-4}\right)$$

Chain reaction

Consider a neutron causing fission in a uranium nucleus producing three neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing nine neutrons. These nine neutrons in turn may produce twenty seven neutrons and so on. A chain reaction is a self propagating process in which the number of neutrons goes on multiplying rapidly almost in a geometrical progression.

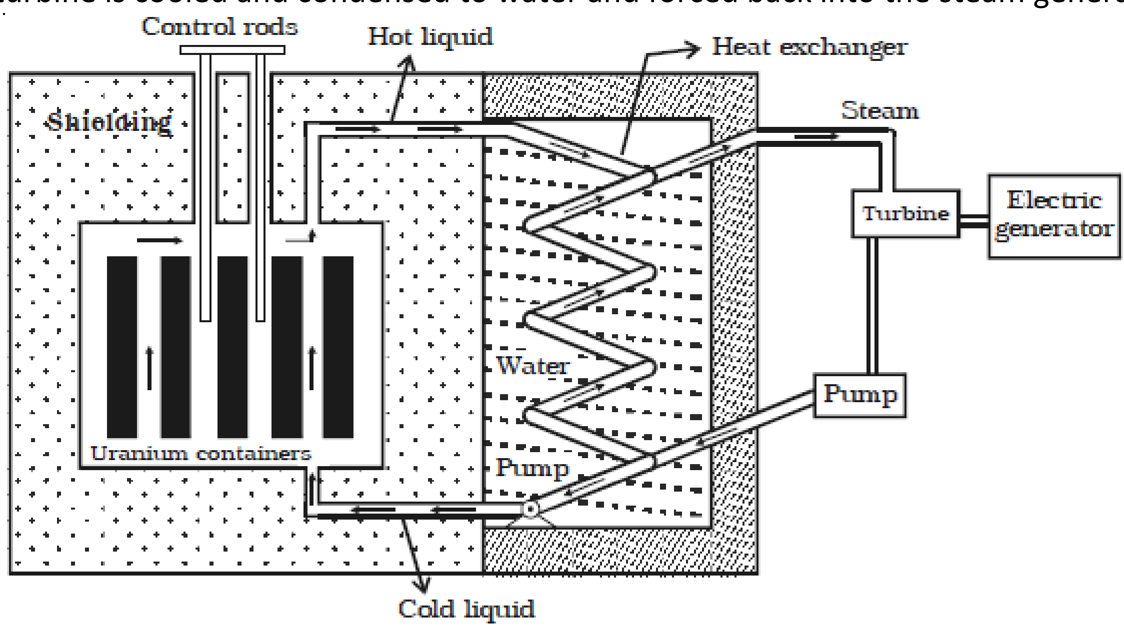
Critical size

Critical size of a system containing a fissile material is defined as the minimum size in which at least one neutron is available for further fission reaction. The mass of the fissile material at the critical size is called critical mass. The chain reaction is not possible if the size is less than the critical size.

Nuclear reactor

A nuclear reactor is a device in which the nuclear fission reaction takes place in a self sustained and controlled manner

The schematic diagram of a nuclear reactor is shown in Fig In such a reactor, water is used both as the moderator and as the heat transfer medium. In the *primary-loop*, water is circulated through the reactor vessel and transfers energy at high temperature and pressure (at about 600 K and 150 atm) to the steam generator, which is part of the *secondary-loop*. In the steam generator, evaporation provides high-pressure steam to operate the turbine that drives the electric generator. The low-pressure steam from the turbine is cooled and condensed to water and forced back into the steam generator



(i) Fissile material or fuel

The fissile material or nuclear fuel generally used is $_{92}U^{235}$. But this exists only in a small amount (0.7%) in natural uranium. Natural uranium is enriched with more number of $_{92}U^{235}$ (2 – 4%) and this low enriched uranium is used as fuel in some reactors. Other than U^{235}, the fissile isotopes U^{233} and Pu^{239} are also used as fuel in some of the reactors.

(ii) Moderator

The function of a moderator is to slow down fast neutrons produced in the fission process having an average energy of about 2 *MeV* to thermal neutrons with an average energy of about 0.025 *eV*, which are in thermal equilibrium with the moderator. Ordinary water and heavy water(D_2O) are the commonly used moderators. A good moderator slows down neutrons by elastic collisions and it does not remove them by absorption. The moderator is present in the space between the fuel rods in a channel. Graphite is also used as a moderator in some countries. In fast breeder reactors, the fission chain reaction is sustained by fast neutrons and hence no moderator is required.

(iii) Neutron source

A source of neutron is required to initiate the fission chain reaction for the first time. A mixture of beryllium with plutonium or radium or polonium is commonly used as a source of neutron.

(iv) Control rods

The control rods are used to control the chain reaction. They are very good absorbers of neutrons. The commonly used control rods are made up of elements like boron or cadmium. The control rods are inserted into the core and they pass through the space in between the fuel tubes and through the moderator. By pushing them in or pulling out, the reaction rate can be controlled. In our country, all the power reactors use boron carbide (B4C), a ceramic material as control rod.

Because of the use of control rods, it is possible that the ratio, *K*, of number of fission produced by a given generation of neutrons to the number of fission of the preceeding generation may be greater than one. This ratio is called the *multiplication factor;* it is the measure of the growth rate of the neutrons in the reactor. For $K = 1$, the operation of the reactor is said to be *critical*, which is what we wish it to be for steady power operation. If *K* becomes greater than one, the reaction rate and the reactor power increases exponentially. Unless the factor *K* is brought down very close to unity, the reactor will become supercritical and can even explode.

In addition to control rods, reactors are provided with *safety rods* which, when required, can be inserted into the reactor and *K* can be reduced rapidly to less than unity.

(v) The cooling system

The cooling system removes the heat generated in the reactor core. Ordinary water, heavy water and liquid sodium are the commonly used coolants. A good coolant must possess large specific heat capacity and high boiling point. The coolant passes through the tubes containing the fuel bundle and carries the heat from the fuel rods to the steam generator through heat exchanger. The steam runs the turbines to produce electricity in power reactors.
Being a metal substance, liquid sodium is a very good conductor of heat and it remains in the liquid state for a very high temperature as its boiling point is about 1000^o C.

(vi) Neutron reflectors
Neutron reflectors prevent the leakage of neutrons to a large extent, by reflecting them back. In pressurized heavy water reactors the moderator itself acts as the reflector.
In the fast breeder reactors, the reactor core is surrounded by depleted uranium (uranium which contains less than 0.7% of $_{92}U^{235}$) or thorium ($_{90}Th^{232}$) which acts as neutron reflector. Neutrons escaping from the reactor core convert these materials into Pu^{239} or U^{233} respectively.

(vii) Shielding
As a protection against the harmful radiations, the reactor is surrounded by a concrete wall of thickness about 2 to 2.5 m.

Nuclear fusion – energy generation in stars

Energy can be released if two light nuclei combine to form a single larger nucleus, a process called *nuclear fusion*.
Some examples of such energy liberating reactions are

$$\mathbf{{}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + \nu + 0.42\ MeV \quad \text{-- (a)}}$$
$$\mathbf{1H^2 + 1H^2 \rightarrow {}_2He^3 + n + 3.27\ MeV \quad \text{---(b)}}$$
$$\mathbf{1H^2 + 1H^2 \rightarrow {}_1He^3 + {}_1H^1 + 4.03\ MeV \quad \text{--- (c)}}$$

In all these reactions, we find that two positively charged particles combine to form a larger nucleus.
It must be realized that such a process is hindered by the Coulomb repulsion that acts to prevent the two positively charged particles from getting close enough to be within the range of their attractive nuclear forces and thus 'fusing'.
The height of this *Coulomb barrier* depends on the charges and the radii of the two interacting nuclei. The temperature at which protons in a proton gas would have enough energy to overcome the coulomb's barrier is about 3×10^9 K.
To generate useful amount of energy, nuclear fusion must occur in bulk matter. What is needed is to raise the temperature of the material until the particles have enough energy – due to their thermal motions alone – to penetrate the coulomb barrier. This process is called *thermonuclear fusion*.

The fusion reaction in the sun is a multi-step process in which hydrogen is burned into helium, hydrogen being the 'fuel' and helium the 'ashes'.
The *proton-proton* (p, p) *cycle* by which this occurs is represented by the following sets of reactions:

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + \nu + 0.42\ \text{MeV} \quad \text{--- (d)}$$
$$e^+ + e^- \rightarrow \gamma + 1.02\ \text{MeV} \quad \text{--- (e)}$$
$${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + \gamma + 5.49\ \text{MeV} \quad \text{--- (f)}$$
$${}_2H^3 + {}_2H^3 \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1 + 12.86\ \text{MeV} \quad \text{---(g)}$$

For the fourth reaction to occur, the first three reactions must occur twice, in which case two light helium nuclei unite to form ordinary helium or nucleus.
the net effect is

$$4\,{}_1H^1 + 2e^- \rightarrow {}_2He^4 + 2\nu + 6\gamma + 26.7\text{MeV}$$

Thus, four hydrogen atoms combine to form an ${}_2He^4$ atom with a release of 26.7 MeV of energy.
Calculations show that there is enough hydrogen to keep the sun going for about the same time into the future. In about 5 billion years, however, the sun's core, which by that time will be largely helium, will begin to cool and the sun will start to collapse under its own gravity. This will raise the core temperature and cause the outer envelope to expand, turning the sun into what is called a *red giant*
If the core temperature increases to 10^8 K again, energy can be produced through fusion once more – this time by burning helium to make carbon. As a star evolves further and becomes still hotter, other elements can be formed by other fusion reactions.
The energy generation in stars takes place via thermonuclear fusion.

Solved Numerical

Q) By the fusion of 1Kg deuterium ($1H^2$) according the reaction
${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 3.27$ Mev, how long can a bulb of 100 W give light?
Molecular wt of deuterium is 2g
Thus number of moles of deuterium in 1kg = 500 moles
Number of nucli of deuterium = $500\times6.02\times10^{23} = 3.01\times10^{26}$
Now Two nucli gives energy of 3.27 MeV = $3.27\times10^6\times1.6\times10^{-19}$ J=5.23×10^{-13} J
Thus 3.01×10^{26} nucli will release energy of

$$\frac{3.01\times10^{26}\times5.23\times10^{-13}}{2} = 7.87\times10^{13}J$$

If a bulb of 100 W glows for t seconds, then energy consumed = 100t J

$$\therefore 100t = 7.87 \times 10^{13}$$

t = 7.874×10^{11} sec

$$t = \frac{7.874 \times 10^{11}}{3.16 \times 10^{7\ s/year}} = 24917\ Yr$$