UNITS, DIMENSION AND MEASUREMENT

- Measurement of large distance (Parallax Method)

  $$D = \frac{b}{\theta}$$

  Here

  D = distance of the planet from the earth.

  θ = parallax angle.

  b = distance between two place of observation

- Measurement of the size of a planet or a star.

  $$\alpha = \frac{d}{D}$$

  Here

  D = distance of planet from the earth,

  d = diameter of planet.

  α = angular diameter of planet.

- Measurement of mass

  The gravitational force on object, of mass m, is called the weight of the object.

  1 amu = 1.66 × 10-27 kg = 1u

- Estimation of Error

  Suppose the values obtained in several measurement of physical quantity

  are a1, a2, ... an . Their arithmetic mean is

  $$\bar{a} = \frac{a_1 + a_2 + ...+ a_n}{n} = \frac{1}{n}\sum_{i}^{n} a_i$$

- Absolute Error -

  $\Delta a_1 = \bar{a} - a_1$

  $\Delta a_2 = \bar{a} - a_2$

  $-------$

  $-------$

  $\Delta a_n = \bar{a} - a_n$

  $\Delta a_1, \Delta a_2 \ldots, \Delta a_n$ called absolute error

- Average absolute error

  $$\Delta a = \frac{|\Delta a_1| + |\Delta a_2| + \ldots\ldots + |\Delta a_2|}{n}$$

- Fractional Error

  $$\delta a = \frac{\Delta a}{a}$$

- Percentage Error

  $$\delta a\% = \frac{\Delta a}{a} \times 100\%$$

- Combination of Errors

  (i) Addition : Z = A + B

  Maximum Absolute Error = ΔA + Δ B

  Maximum Relative Error=

  $$\frac{\Delta A + \Delta B}{A + B}$$

  Maximum Percentage Error =

  $$\left(\frac{\Delta A + \Delta B}{A + B}\right) \times 100$$

  (ii) Subtractions : Z = A – B

  Maximum Absolute Error = ΔA + Δ B

  Maximum Relative Error = $\frac{\Delta A + \Delta B}{A - B}$

Maximum Percentage Error =
$\left(\frac{\Delta A+\Delta B}{A-B}\right)\times 100$

(iii) Multiplication Z = A × B

Maximum Absolute error = AΔB + BΔA

Maximum Relative Error=

$$\frac{\Delta A}{A}+\frac{\Delta B}{B}$$

Maximum Percentage Error =

$$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$$

(iii) Division Z = A/B

Maximum Absolute error =

$$\frac{B\Delta A + A\Delta B}{B^2}$$

Maximum Relative Error =

$$Z=\frac{A}{B}$$

$$\frac{\Delta Z}{Z}=\frac{\Delta A}{A}+\frac{\Delta B}{B}$$

(iv) Power Z = A^n

Maximum Absolute Error : $n\,A^{n-1}\Delta A$

Maximum Relative Error

$$\frac{\Delta Z}{Z}=n\frac{\Delta A}{A}$$

Maximum Percentage Error=

$$n\frac{\Delta A}{A}\times 100$$

- Rule for determining number of significant figures

  (i) All the non - zero digits are significant

  (ii) All the zeros between two non zero digits are significant no matter where the decimal point is it at all.

  (iii) If the number is less then 1 then zeros on the right of decimal point but to the left of the first non - zero digit are not significant.

  (iv) In a number without decimal point the zeros on the right side of the last non zero digit are not significant.

- Dimensions and Dimensional formulas.

  (i) The expression of a physical quantity with appropriate powers of M, L, T, K, A etc is called the dimensional formula of that physical quantity.

  (ii) The power of exponents of M, L, T, K, A are called dimensions of that quantity.

- Some important units of distance

  1fermi (fm) =10^{-15}m

  1Å = 10–10m

  1AU =1.496×1011m

  1light year = 9.46×1015m

  1par sec = 3.08×1016m

- Conversion of one System of units into another

  $$n_2 = n_1 \frac{u_1}{u_2}$$

  $$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

  Units used : Every quantity must be expressed in its absolute units only

- Principle of homogeneity of dimensions: Equating the power of M,L,T on either side of the equation OR power of dimension either side must be same
- Quantities having same dimensions/ units

  (i) Frequency, angular frequency, angular velocity, velocity gradient : $M^0 L^0 T^{-1}$

  (ii) Work, Internal energy, P.E. , K.E., torque, moment of force : $M^1 L^2 T^{-2}$

  (iii) Pressure, Stress, Young's modulus, Modulus of rigidity, Energy density : $M^1 L^{-1} T^{-2}$

  (iv) Mass and Inertia : $M^1 L^0 T^0$

  (v) Momentum and Impulse : $M^1 L^1 T^{-1}$

  (vi) Acceleration, g, gravitational intensity : $M^0 L^1 T^{-2}$

  (vii) Thrust, Force, Weight, Energy radiant : $M^1 L^1 T^{-2}$

  (viii) Angular momentum and Planck's constant (h) : $M^1 L^2 T^{-1}$

  (ix) Surface tension, Surface energy (energy per unit area), force gradient, Spring constant : $M^1L^0T^{-2}$

  (x) Strain, Refractive index, relative density, angle, solid angle, distance gradient, relative permeability, relative permitivity,: $M^0 L^2 T^{-2}$

  (xi) Thermal capacity, gas constant, Boltzmann constant and entropy : $M^1 L^2 T^{-2} K^{-1}$

  (xii) L/R, $\sqrt{(LC)}$ and RC : $M^0 L^0 T^1$

  here R : resistance; C : capacitance; L : inductance

MOTION IN ONE DIMENSION

- Distance

  (i) It is the length of actual path traversed by a body during motion in a given interval of time

  (ii) Distance is a scalar quantity

  (iii) Value of distance travelled by moving body can never be zero or negative.

- Displacement

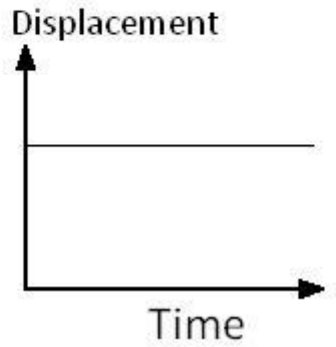
  (i) Displacement of a body in a given time is defined as the change in the position of the body in the particular direction during that time

  (ii) Displacement is vector quantity.

  (iii) The value of displacement can be zero or negative or positive

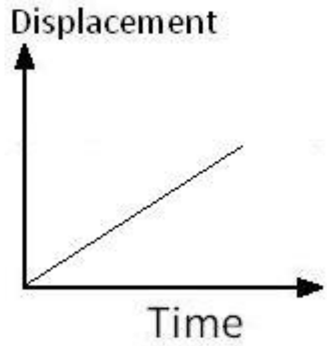
  (iv) The value of displacement can never be greater than the distance travelled.

- Displacement-time graph of various types of motion of a body (i) For stationary body



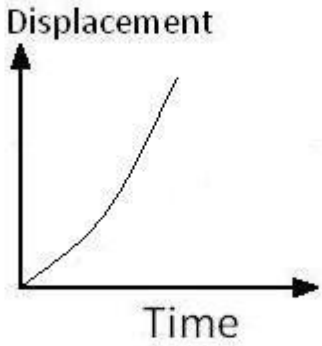
Slope of straight line (representing instantaneous velocity) is zero.

(ii) When body is moving with a constant velocity, or acceleration is zero straight line inclined to time axis



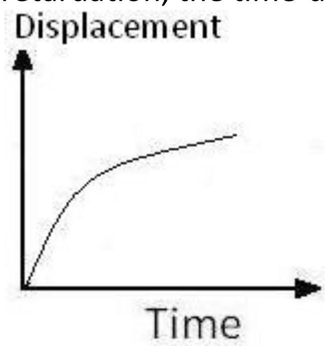
Greater is the slope of straight line higher is the velocity

(iii) When body is moving with a constant positive acceleration, the time-displacement curve with bend upwards



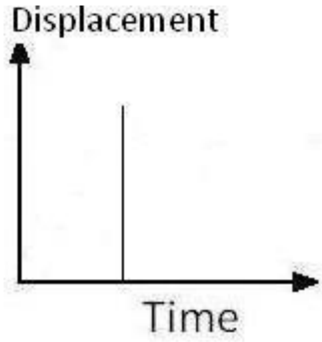
The slope of time-displacement curve increases with time or velocity is increasing with time

(iv) When body is moving with a constant negative acceleration or constant retardation, the time-displacement curve with bend downwards



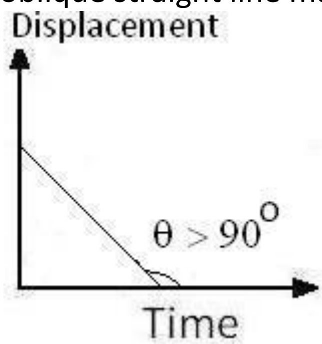
The slope of time-displacement curve (i.e. instantaneous velocity) decreases with time.

(v) When a body is moving with infinite velocity, the time - displacement graph is parallel to displacement axis.



Such motion of a body is never possible

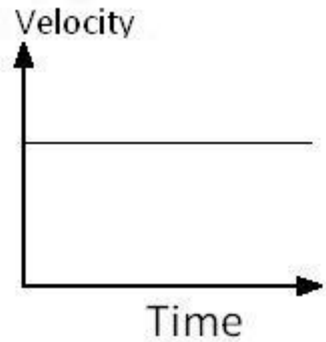
(vi) When body returns back towards the original point of reference while moving with uniform negative velocity, the time displacement graph is an oblique straight line making angle θ > 90°



Displacement of the body decreases with time with respect to the reference point, till it becomes zero

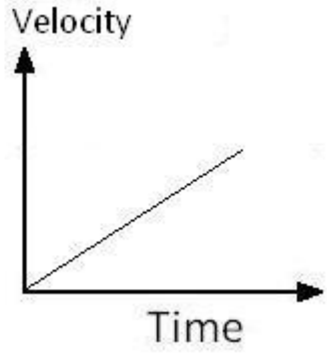
- Velocity - time graph o various types of motion of a body

(i) When a body is moving with a constant velocity, the velocity - time graph is a straight line., parallel to time axis

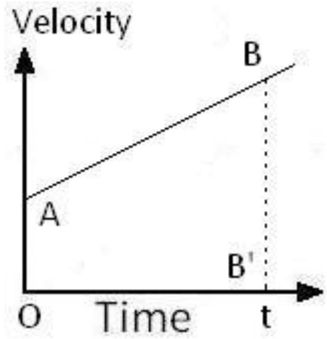


The slope of the graph , representing the instantaneous acceleration is zero

(ii) When a body is moving with a constant acceleration and its initial velocity is zero, the velocity time graph is an oblique straight line, passing through origin



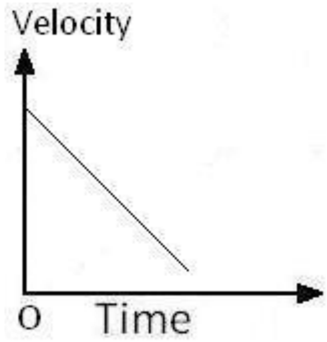
(iii) When a body is moving with constant speed with a constant acceleration and its initial velocity is not zero, the velocity - time graph is an oblique straight line not passing through origin



Here OA represents the initial velocity of the body

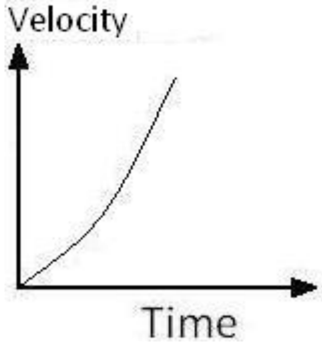
The area enclosed by the velocity-time graph with time axis represents the distance travelled by the body

(iv) When a body is moving with a constant retardation and its initial velocity is not zero, the velocity - time graph is an oblique straight line, not passing through origin



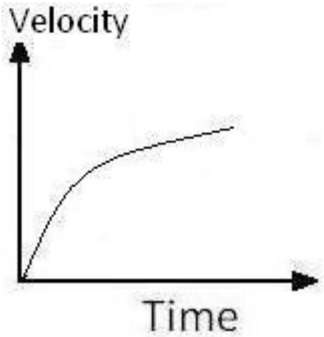
The slope of the line is negative indicating retardation

(v) When a body is moving with increasing acceleration , the velocity - time graph is a curve with bend upwards



The slope of the graph increases with time. or instantaneous acceleration increases with time

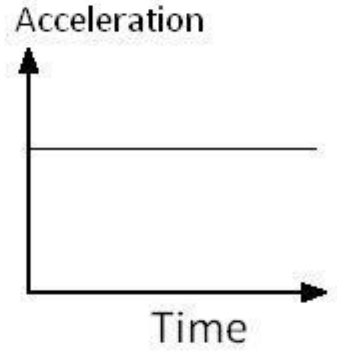
(vi) When a body is moving with decreasing acceleration, the velocity - time graph is a curve with bend downwards



The slope of velocity - time graph decreases with time. or instantaneous acceleration decreases with time

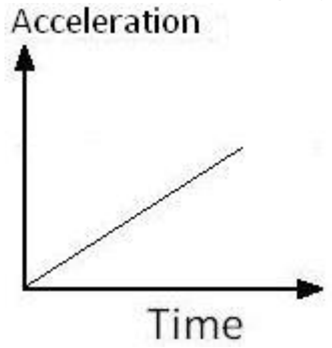
- Acceleration - time graph of various types of motion of body

(i) When a body is moving with constant acceleration, the acceleration - time graph is a straight line, parallel to time axis



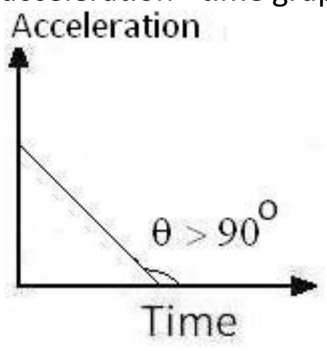
The area enclosed by acceleration - time graph for the body for the given time gives change in velocity of the body for given time

(ii) When a body is moving with constant increasing acceleration, the acceleration - time graph is straight line



The body is moving with positive acceleration

- When a body is moving with constant decreasing acceleration , the acceleration - time graph is straight line



The body is moving with negative acceleration and slope of straight line makes an angle $\theta > 90°$ with time axis. Or slope of the line is negative

- IMPORTANT FORMULAS

(i)Equations of motion

u : initial velocity, v : final velocity , a : acceleration,

t : time period, S: displacement

$$S = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$S = \left(\frac{v+u}{2}\right)t$$

$$a = \frac{v-u}{t}$$

(ii) Free fall

h: height, g = gravitational acceleration , final velocity :v, t: time period

$$v = gt$$

$$v^2 = 2gh$$

(iii) Object thrown up

h: height, g = gravitational acceleration , u : initial velocity(negative) , t: time period

$$u = \sqrt{2gh}$$

$$h = \frac{u^2}{2g}$$

(iv) Distance travelled during last n sec while body is moving up = distance travelled during first n second of free fall

(v) Distance travelled in nth second

$$S_n = u + \frac{a}{2}(2n-1)$$

(vi) If time period for two different section is same and v1 and v2 are the velocities for two sections then average velocity <v> is

$$< V > = \frac{V_1 + V_2}{2}$$

(vii) If an object covers distance 'x' with constant speed of v1 and same distance with constant speed of v2then average speed of <v> is

$$< V > = \frac{2V_1V_2}{V_1+V_2}$$

(viii) Starting from position of rest particle moves with constant accelerates +α reaches maximum velocity v_{max} then particle moves with constant decelerated β and become stationary . total time elapsed during this is t , then maximum velocity of particle is v_{max} is

$$V_{max} = \left(\frac{\alpha\beta}{\alpha + \beta}\right) t$$

- Calculus

  S: displacement , v : instantaneous velocity , a : acceleration

$$v = \frac{ds}{dt}$$

$$a = \frac{dV}{dt}$$

- if V_A is magnitude of velocity of object A and V_B is magnitude of velocity of object B with respect to stationary observer then

  (i) if both objects are moving in same direction, velocity of A with respect to B,

  $V_{AB} = V_A - V_B$

  (ii) If both objects are approaching, velocity of A with respect to B ,

  $V_{AB} = V_A + V_B$

MOTION IN TWO AND THREE DIMENSIONS

- Vector

(i) If vector $\overrightarrow{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then magnitude of vector is $\left|\vec{A}\right| = \sqrt{x^2 + y^2 + z^2}$

(ii) If Vector $\overrightarrow{A} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$

and vector $\overrightarrow{B} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$

(a) are equal then

$x_1 = x_2$

$y_1 = y_2$

$z_1 = z_2$

(b) If $\vec{R} = \overrightarrow{A} + \vec{B}$ then

$$\overrightarrow{R} = (x_1 + x_2)\hat{\imath} + (y_1 + y_2)\hat{\jmath} + (z_1 + z_2)\hat{k}$$

(iii) If If $\overrightarrow{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is vector then

(a) vector is making angle α with positive x-axis then

$$cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}} = \frac{x}{|\vec{A}|}$$

(b) Vector is making angle β with positive y-axis then

$$cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}} = \frac{y}{|\vec{A}|}$$

(c) Vector makes angle of γ with positive z-axis then

$$cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z}{|\vec{A}|}$$

(d) We can multiply and divide vector by scalar quantity but we can not divide vector by another vector

(d) $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

(iv) If $\overrightarrow{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is vector then unit vector **n** is

$$\hat{n} = \frac{\vec{A}}{|\vec{A}|}$$

- Addition of two vectors

  (a) Only vectors of same nature can be added

  (b) The addition of two like vectors $\overrightarrow{A}$ and $\vec{B}$ is a resultant vector $\vec{\boldsymbol{R}}$ where

$$R = (A^2 + B^2 + 2ABcos\theta)^{\frac{1}{2}}$$

$$tan\beta = \frac{Bsin\theta}{A + Bsin\theta}$$

  Here θ is the angle between vector $\overrightarrow{A}$ and vector $\vec{B}$

  β is the angle between resultant vector $\vec{R}$ and vector $\vec{A}$

  (c) Vector addition is commutative

- (d) Vector addition is associative
- (e) $|\vec{R}|$ is maximum when θ = 0 and minimum when θ = 180
- Subtraction of two vectors

  (i) Only vectors of same nature can be subtracted

  (ii) The subtraction of two like vectors $\overrightarrow{A}$ and $\vec{B}$ is a resultant vector $\vec{R}$ where

$$R = (A^2 + B^2 - 2ABcos\theta)^{\frac{1}{2}}$$

$$tan\beta = \frac{Bsin\theta}{A - Bsin\theta}$$

  Here θ is the angle between vector $\overrightarrow{A}$ and vector $\vec{B}$

  β is the angle between resultant vector **R** and vector **A**

  (iii) Vector addition is NOT commutative (iv) Vector addition is NOT associative

  (v) Magnitude of resultant of the vector subtraction is equal to that of vector addition if angle between $\overrightarrow{A}$ and $\vec{B}$ is 90°

- Dot product or scalar of two vectors $\overrightarrow{A}$ and $\vec{B}$

  (i) $\overrightarrow{A} \cdot \vec{B}$ = ABcosθ, where θ is the angle between vector $\overrightarrow{A}$ and vector $\vec{B}$

(ii) Dot product of two vector is scalar

(iii) Dot product of two parallel vectors is maximum value $\overrightarrow{A} \cdot \vec{B}$ = AB

(iv) Dot product of perpendicular vectors is zero in value $\overrightarrow{A} \cdot \vec{B}$ = 0

A unit vector is a unit less and dimensionless vector. Its magnitude is one and it represent direction only.

(v) Dot product of two vector is commutative

(vi) $\hat{\imath}.\hat{\imath} = \hat{\jmath}\,.j = \hat{k}\,.\,\hat{k} = 1$

(vii) $\hat{\imath}\,.\,\hat{\jmath} = \hat{\jmath}\,.\,\hat{k} = \hat{k}\,.\,\hat{\imath} = 0$

(viii) In cartesian co-ordinate

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

- Cross product or vector product of two vectors $\hat{A}$ and $\widehat{B}$

  (i) $\vec{A} \times \vec{B} = \vec{C} = \text{ABsin}\,\hat{n},$

  Here θ is the angle between vector $\hat{A}$ and $\widehat{B}$, $\hat{n}$ is the unit vector of resultant vector $\vec{C}$

  Direction of $\vec{C}$ can be determined by right hand screw rule.

  (ii) Magnitude of the cross product of two vector is equal to

  (a) Twice the area of a triangle whose two sides are represented by two vectors.

  (b) Area of parallelogram whose two sides represented by two vectors.

  (iii) Cross product of two parallel vectors is zero.

  (iv) Cross product of two perpendicular vector is maximum.

  (v) Cross product of two vectors is ant commutative i.e $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

  (vi) Cross product of vectors is distributive.

  (vii) For cross product

$$\widehat{\imath} \times \hat{\imath} = \widehat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

  (a) $\hat{\imath} \times \hat{\jmath} = \hat{k}$ and $\hat{\jmath} \times \hat{\imath} = -\hat{k}$

(b) $\hat{j} \times \hat{k} = \hat{\imath}$ and $\hat{k} \times \hat{j} = -\hat{\imath}$

(c) $\hat{k} \times \hat{\imath} = \hat{j}$ and $\hat{\imath} \times \hat{k} = -\hat{\imath}$

-
- (viii) In Cartesian co-ordinates

$$\vec{A} \times \vec{B} = \left(A_x\hat{\imath} + A_y\hat{j} + A_z\hat{k}\right) \times \left(B_x\hat{\imath} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{j} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

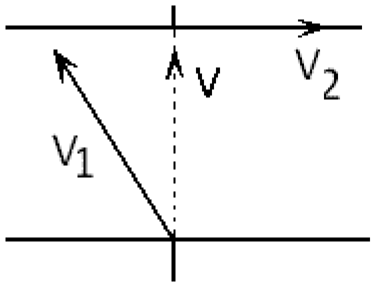
$$\vec{A} \times \vec{B} = \left(A_yB_z - A_ZB_y\right)\hat{\imath} - (A_xB_z - A_ZB_x)\hat{j} + \left(A_xB_y - A_yB_y\right)\hat{k}$$

- Tensors: Those physical quantities which have no specified direction but have different values in different directions are called tensors.
  for example: Moment of inertia, stress, strain, density, refractive index, electrical conductivity etc.. Which are normally scalar but in anisotropic medium they assume different values in different directions so becomes tensors.

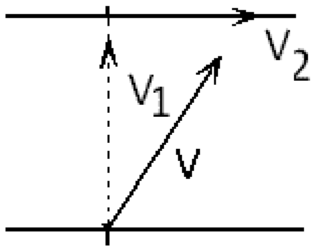
- When a boat tends to cross a river of width along a shortest path
  It should be rowed upstream making angle θ with the perpendicular direction of the river flow
  so that resultant velocity $\overrightarrow{\boldsymbol{v}}$ of , velocity of boat $\vec{\boldsymbol{v}}_1$ and velocity of flow of river $\vec{\boldsymbol{v}}_2$ may act perpendicular to the direction of river flow. in this case

$$sin\theta = \frac{v_2}{v_1}$$

$$v = \sqrt{v_1^2 - v_2^2}$$



- When a boat tends to cross a river in shortest time



The boat should go along the direction, perpendicular to the direction of river flow. then the boat will be going along the direction of resultant velocity $\vec{v}$ of velocity of boat $\overrightarrow{v_1}$ and velocity of flow of river $\overrightarrow{v_2}$. If $\vec{v}$ making an angle θ with the direction of $\overrightarrow{v_1}$, If S is the width of river and time t is time of crossing then

$$tan\theta = \frac{v_2}{v_1}$$

$$v = \sqrt{v_1^2 - v_2^2}$$

$$t = \frac{S}{v_1}$$

- Relative velocity

If $\vec{V}_{AO}$ is velocity vector of object A with respect to observer O

If $\vec{V}_{BO}$ is velocity of object B with respect to observer O

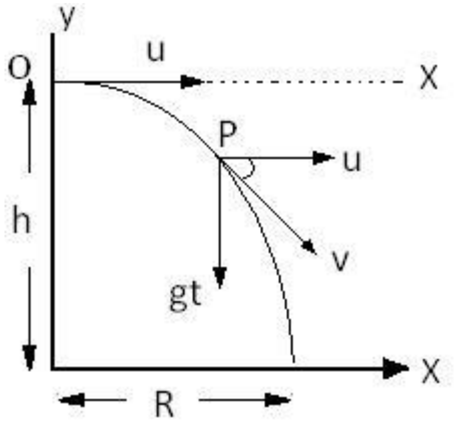
IF $\vec{V}_{AB}$ is the velocity of A with respect to B then

$\vec{V}_{AB} = \vec{V}_{AO} - \vec{V}_{BO}$ OR

$\vec{V}_{AB} = \vec{V}_{AO} + \vec{V}_{OB}$ (as $- \vec{V}_{BO} = \vec{V}_{OB}$)

- Projectile projected with initial velocity u

A) Horizontal projection from a height h angle of projection is zero



(i) Equation for path y =

$$y = \frac{1}{2}\frac{gx^2}{u^2}$$

(ii) Velocity of projectile at any instant t is v =

$$v = \sqrt{u^2 - g^2t^2}$$

(iii) Direction of velocity **v** with the horizontal =

$$\beta = tan^{-1}\left(\frac{gt}{u}\right)$$

(iv) Time of flight T =

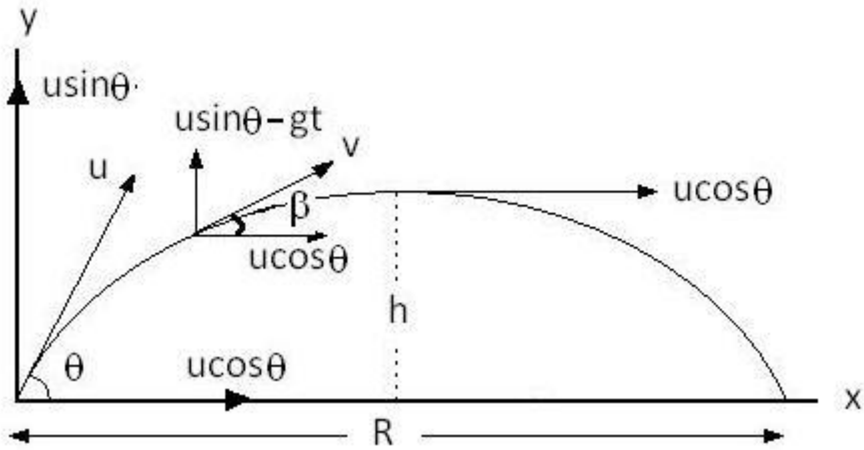
$$T = \sqrt{\frac{2h}{g}}$$

(v) Horizontal Range R =

$$R = u\sqrt{\frac{2h}{g}}$$

(vi) Maximum Height H = h

B) Angular projection of projectile making an angle θ with the horizontal direction



(i) Equation for path

$$y = xtan\theta - \frac{gx^2}{2(ucos\theta)^2}$$

(ii) Velocity of projectile at any instant t is

$$v = \sqrt{u^2 + g^2t^2 - 2ugtsin\theta}$$

(iii) Direction of velocity **v** with the horizontal =

$$\beta = tan^{-1}\left(tan\theta - \frac{gt}{ucos\theta}\right)$$

(iv) Time of flight T =

$$T = \frac{2usin\theta}{g}$$

(v) Horizontal Range R =

$$R = \frac{u^2 sin2\theta}{g}$$

(vi) Maximum Height H =

$$H = \frac{u^2 sin^2\theta}{2g}$$

(vii) At highest point of projectile path, the velocity and acceleration are perpendicular to each other.

(viii) At highest point of projectile motion, the angular momentum of projectile is

L = momentum of projectile × maximum height

$$L = m\,u\,cos\theta \times \frac{u^2 sin^2\theta}{2g}$$

(ix) The particle returns to the ground at the same angle and with the same speed with which it was projected.

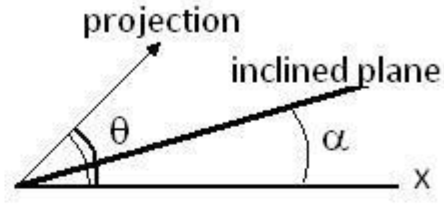
(x) if θ_1 and θ_2 are the angle of projection have same range then

$$\theta_1 + \theta_2 = \pi/2$$

C) Relation between Maximum height and Range

4H = R tanθ

D) Projectile along the inclined plane α is angle of inclination with horizontal, θ is the angle of projection with horizontal



- (i) Range of projectile along the inclined plane is =

$$R' = \frac{u^2}{g cos^2 \alpha}[sin(2\theta - \alpha) - sin\alpha]$$

(ii) Time of flight on an inclined plane

$$T' = \frac{2u sin(\theta - \alpha)}{g}$$

(iii) The angle at which the horizontal range on the inclined plane becomes maximum is given by

$$\theta = \frac{\pi}{4} + \frac{\alpha}{2}$$

(iv) Maximum range on inclined plane is

$$R'_{max} = \frac{u^2}{g(1 + sin\alpha)}$$

- Centripetal force

(i) The centripetal force does not increases the kinetic energy and angular momentum of the particle moving in circular path, hence work done is zero

(ii) When body moves with constant angular velocity, then its acceleration always acts perpendicular to the velocity. Acceleration is towards the centre of the circle along which body is moving it is called centripetal acceleration. Whose magnitude is given by

$$a_c = \frac{v^2}{r} = r\omega^2$$

(iii) Force required to keep the body in circular path with constant angular velocity is centripetal force and is given by

$$F = \frac{mv^2}{r} = mr\omega^2$$