

HEAT AND THERMODYNAMICS

- First law of thermodynamics

$$dQ = dU + dW$$

Where dQ is the small amount of heat energy given to the system, dU is small increase in internal energy of system and dW is the small external work done by the system.

(i) For isothermal process $T = \text{constant}$

$$\therefore dT = 0 \text{ and } dU = 0$$

$$\therefore dQ = dW = p(dV)$$

(ii) In adiabatic process, $dQ = 0 \therefore dW = -dU$

(iii) In an isochoric process $V = \text{constant}$

$$\therefore dv = 0, dW = p(dV) = 0$$

$$dQ = dU$$

- sign convention used in thermodynamics

(i) Heat absorbed by the system is taken as positive and heat rejected by the system is taken as negative

(ii) When temperature of the system rises, its internal energy increases ΔU is taken as positive

when temperature of the system falls, its internal energy decreases, ΔU is taken negative

(iii) When a gas expands, work is done by the system. It is taken as positive, When a gas is compressed, work is done on the system. It is taken negative

- Work done at constant temperature

$$W = \mu RT \ln \left(\frac{V_f}{V_i} \right)$$

$PV = \text{Constant}$ (Isothermal process)

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- Work done at constant temperature and at constant pressure $W = P [V_f - V_i]$
- Work done during adiabatic process, when gas goes from initial state (P_1, V_1, T_1) to (P_2, V_2, T_2)

is

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$W = \frac{\mu R (T_1 - T_2)}{\gamma - 1}$$

$PV^\gamma = \text{constant}$ (for adiabatic process)

- The ratio of heat Q supplied to a body to a change in its temperature ΔT is called heat capacity H_c of the body.

$$H_c = Q / \Delta T$$

SI unit of HC is JK^{-1} or cal/K

Heat capacity of a body depends on the material of the body as well as on its mass.

- The quantity of heat required per unit mass for unit change in temperature of a body is called the specific heat of the material of the body

$$C = Q / m\Delta T$$

Unit is $\text{cal g}^{-1} \text{K}^{-1}$ or $\text{Jkg}^{-1} \text{K}^{-1}$

- Molar specific heat : Amount of heat required to increase the temperature of one mole of gas by 1K (or 1°C) is called molar specific heat

(i) Molar specific heat at constant volume C_v

(ii) Molar specific heat at constant pressure C_p

(iii) $C_p - C_v = R$

(iv) $C_p C_v = \gamma$

if f is degree of freedom then

$$\gamma = 1 + (2/f)$$

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for mono-atomic $f = 3$

for diatomic rigid rotor $f = 5$ (eg. at moderate temperature O₂)

for diatomic molecules $f = 7$ (example CO)

Energy associated with each degree of freedom = $\frac{1}{2} k_B T$

(k_B = Boltzmann's constant = 1.38×10^{-23})

- for $f = 3$, $C_v = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$
- for $f = 5$, $C_v = 5R/2$, $C_p = 7R/2$, and $\gamma = 7/5$
- for $f = 7$, $C_v = 7R/2$, $C_p = 9R/2$, and $\gamma = 9/7$

- Internal energy of gas

$$E_i = \mu f R T / 2$$

- Efficiency of heat engine

$$\eta = 1 - \frac{Q_r}{Q_a}$$

Here Q_a is heat absorbed, and Q_r is heat released in to heat sink

Efficiency of heat engine in terms of temperature

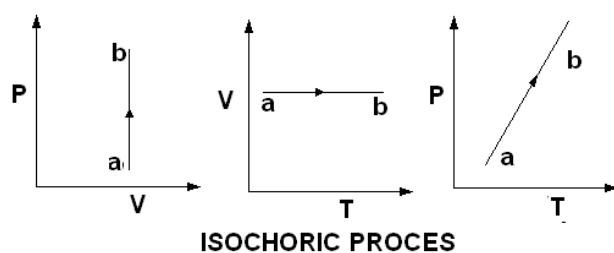
$$\eta = 1 - \frac{T_{low}}{T_{high}}$$

here T_{low} is lower temperature of sink and T_{high} is higher temperature of source

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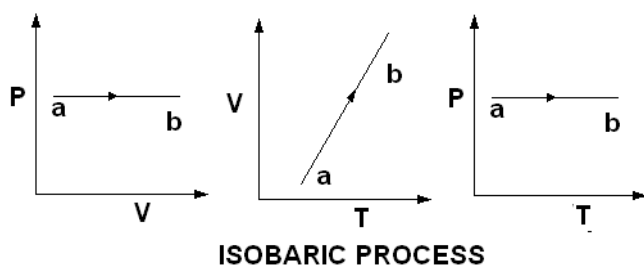
GRAPHS:

A) Work done is zero for isochoric process (Volume remains constant)
The P-V, V-T and P-T diagrams for isochoric process will be like curves given below



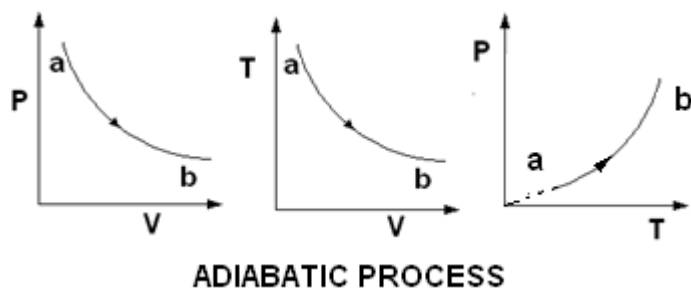
B) isobaric process (Pressure remains constant)

The P-V, V-T and P-T diagrams for isobaric process will be like curves given below

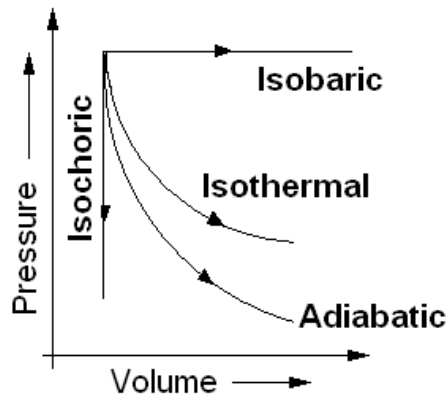


C) Adiabatic process (No transfer of material energy from system)

The P-V, T-V and P-T diagrams for adiabatic process will be lie the curves given below

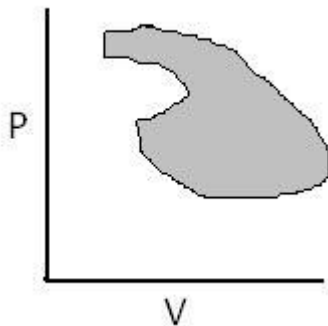


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D) Work = area under the curve on a P-V diagram -- even if the process is not isobaric.

work = Shaded Area



I) S.H.M

- IN S.H.M the displacement of particle from its mean position at an any instant is given by $y = a \sin(\omega t + \phi_0)$

Here a = amplitude of S.H.M., ω = angular frequency and ϕ_0 is initial phase of the particle The direction of displacement is always away from the mean position, whether the particle is moving away from or coming towards the mean position.

- Velocity of S.H.M. at an instant $v = a\omega \cos(\omega t + \phi_0)$ or

$$v = \pm \omega \sqrt{a^2 - y^2}$$

Direction of velocity is either towards or away from the mean position

Acceleration of S.H.M is $A = -\omega^2 y$.

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Here negative sign shows that acceleration is always directed towards the mean position in S.H.M.

- Restoring force on the particle in S.H.M. at an instant is $F = -m\omega^2y$
- Kinetic energy of a particle at an instant in S.H.M is

$$E = \frac{1}{2}mv^2$$

$$E = \frac{1}{2}m\omega^2(a^2 - y^2)$$

Kinetic energy is maximum at the mean position and is minimum at the extreme position

- Potential energy of a particle at an instant in S.H.M. is

$$U = \frac{1}{2}m\omega^2y^2$$

Potential energy is maximum at the extreme position and is minimum at the mean position

- Total energy of the particle in S.H.M. is constant at all instant = $\frac{1}{2}m\omega^2a^2$
- In SHM
 - (i) Velocity leads the displacement by $\pi/2$
 - (ii) Acceleration leads the displacement by phase π
 - (iii) Acceleration leads the velocity by phase $\pi/2$

- Periodic time

$$T = 2\pi \sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

- Pendulum

(i) Simple pendulum periodic time

$$T = 2\pi \sqrt{\frac{l}{g}}$$

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This relation is valid only when $l \ll R$ (radius of earth)

(a) If l is comparable to R the time period of simple pendulum is given by

$$T = 2\pi \sqrt{\frac{R}{\left(1 + \frac{R}{l}\right)g}}$$

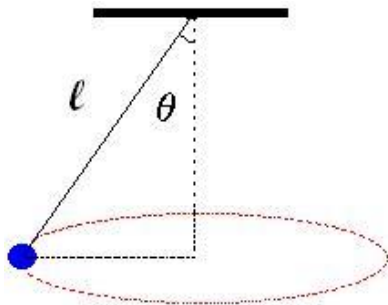
$$T = 2\pi \sqrt{\frac{lR}{(l+R)g}}$$

(b) When $l = \infty$ the $R/l = 0$, and $T = 84.6$ minutes

Thus maximum value of time period of vibration of simple pendulum is 84.6 minutes

(ii) In freefalling lift time period of simple pendulum is ∞ . That is pendulum does not oscillate at all

(iii) When the bob of simple pendulum describes a horizontal circle and length of simple pendulum makes an angle of θ with vertical, then time period of rotational motion is



$$T = 2\pi \sqrt{\frac{l \cos \theta}{g}}$$

(iv) If a simple pendulum of length l is suspended from a roof of compartment of train moving down an inclined plane of inclination θ , time period

$$T = 2\pi \sqrt{\frac{l}{g \cos \theta}}$$

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(v) In case of simple pendulum

(a) graph between l and T is a parabola

(b) graph between l and T^2 is a straight line

(vi) If the temperature of simple pendulum increases by $d\theta$

then %Increase in time = $50\alpha d\theta$

here α is coefficient of linear expansion

(vii) If acceleration due to gravity increases $x\%$, then time period increases by

$(x/2)\%$

(viii) If length of simple pendulum increases by $y\%$ then , then its time period

increase by $(y/2)\%$

- If a ball of radius r oscillates in bowl of radius R ($R > r$) , then the time period of oscillation of the ball is

$$T = 2\pi \sqrt{\frac{R-r}{g}}$$

- If a disc of radius r oscillates about a point at its rim, then its time period of oscillation is

$$T = 2\pi \sqrt{\frac{r}{g}}$$

- if body oscillates with time T_1 and T_2 under the influence of force F_1 and F_2 respectively, then time period of oscillation of bob under resultant force of two forces is given by

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

- Spring

(i) If mass m is suspended from a spring of force constant k , then time period

$$T = 2\pi \sqrt{\frac{m}{k}}$$

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(ii) If the length of spring of spring constant k is made n time then its effective spring constant = k/n and when same mass m is suspended then time period

$$T' = \sqrt{n} T$$

(iii) If spring of spring constant k is divided in n equal parts then spring constant of each part becomes nk . if same mass M is suspended then periodic time $T' = T/\sqrt{n}$

(iv) if spring of length l and spring constant k , is cut in two pieces of length l_1 and l_2 of spring constant k_1 and k_2 such that $l_1 = nl_2$, where n is integer then

$$k_1 = \frac{k(n+1)}{n}$$

and

$$k_2 = k(n+1)$$

(v) If two springs of spring constant k_1 and k_2 are connected in series, then spring constant of combination is

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

Time period of combination

$$T = 2\pi \sqrt{\frac{M(k_1 + k_2)}{k_1 k_2}}$$

(vi) If two springs of spring constant k_1 and k_2 are connected in parallel, then spring constant of combination is

$$k = k_1 + k_2$$

Time period of combination

$$T = 2\pi \sqrt{\frac{M}{k_1 + k_2}}$$

(vii) If two masses of mass m_1 and m_2 are connected at the two ends of the spring constant k , then time period of their oscillation is given by

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$$T = 2\pi \sqrt{\frac{m_1 m_2}{(m_1 + m_2)k}}$$

(viii) If a wire of length l , area of cross-section a , Young's modulus Y is stretched by suspending a mass m , then the period of vibration

$$T = 2\pi \sqrt{\frac{ml}{Ya}}$$

- Differential equations

(i) Without damping

$$\frac{d^2 y(t)}{dt^2} + \omega^2 y(t) = 0$$

(ii) Damped oscillation

$$m \frac{d^2 y(t)}{dt^2} + b \frac{dy(t)}{dt} + ky(t) = 0$$

b is damping constant has SI unit kg/second

solution of this equation

$$y(t) = A_t \sin(\omega' t - \phi)$$

$$A_{(t)} = A e^{-bt/2m}$$

$A(t)$ is amplitude of damped oscillation

Angular frequency

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(iii) Forced Oscillation

$$\frac{d^2 y(t)}{dt^2} + \frac{b}{m} \frac{dy(t)}{dt} + \frac{k}{m} y(t) = \frac{F_0}{m} \sin \omega t$$

The amplitude for forced oscillation is

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$$A = \frac{F_0}{[m^2(\omega_0^2 - \omega^2) + b^2\omega^2]^{1/2}}$$

RAY AND WAVE OPTICS

- When two mirrors are kept facing each other at an angle θ and an object is placed in between them, multiple images of the object are formed as a result of multiple successive reflection "n"

$$n = \left(\frac{360}{\theta} \right) - 1$$

- If the plane mirror is rotated in the plane of incidence by an angle θ , then the reflected ray rotates by an angle of 2θ , the normal rotates by an angle of θ while incident ray remains fixed.
- Minimum size of the mirror required to see full size image of oneself
Least size of the mirror required is half the height of the observer.
- Minimum size of the plane mirror fixed on the wall of a room in which an

observer at the centre of the room can see the full image of the wall behind him

The least size of the mirror required is one-third the height of the wall

- Deviation produced on one reflection $\delta = (180 - 2i)$
Also $\delta = 2\theta$; where θ is the glancing angle i.e. the angle made between the incident ray and the surface of the plane mirror
- Spherical mirror

(i) Conjugate distance

u = object distance, v = image distance, f = focal length

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

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(ii) $R =$ radius of curvature then focal length $f = R/2$

(iii) Newton's formula for concave mirror

x_1 is the object distance from the focus and x_2 is image distance from focus

$$f = x_1 x_2$$

Linear magnification $m = v/u$

(iv) a real magnification

$$m^2 = \frac{v^2}{u^2} = \frac{\text{Area of image}}{\text{Area of object}}$$

- Snell's law

(i) $i =$ angle of incidence, $r =$ angle of refraction then

$$\frac{\sin i}{\sin r} = \text{CONS.}$$

constant is known as refractive index of medium

(ii) if v_1 velocity of light in medium one and v_2 is the velocity of light in medium two then

Light is entering in medium two from medium one

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

(iii) If n_1 is refractive index of medium one and n_2 is the refractive index of medium two then

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

Note when light passes from one transparent medium to another transparent medium its frequency do not change

Refractive index of medium two with respect to medium one is denoted by n_{21}

- Virtual height /depth when object and observer are in two different medium
 h_i is apparent depth of image

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n_1 is refractive index of observer's medium

h_o is actual depth of object

n_2 is refractive index of object medium

$$\frac{h_i}{h_o} = \frac{n_1}{n_2}$$

(ii) If a beaker is filled with immiscible liquids of refractive index μ_1, μ_2, μ_3 , etc.

filled up to heights d_1, d_2, d_3 ...etc

Then apparent depth is :

$$= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + \dots$$

Lateral shift , rectangular slab

t = thickness of glass slab, i = angle of incidence , μ = refractive index of glass slab

$$\delta = ti \left[1 - \frac{1}{\mu} \right]$$

- Total internal reflection

For total internal reflection to take place, it is necessary that ;

(i) The ray should travel from denser medium to rarer medium

(ii) The angle of incidence should be more than critical angle

n_r = refractive index of rarer medium and n_d = refractive index of denser medium

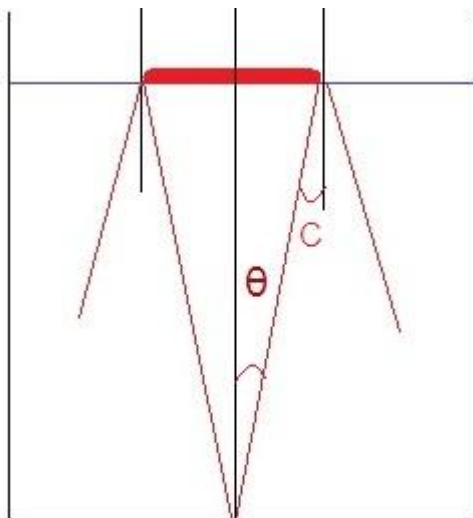
C = critical angle

$$\sin C = \frac{n_r}{n_d}$$

Critical angle increases with increase in temperature

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- Field of vision of fish



h = depth of fish in water r = radius of circular path observed by fish

μ = refractive index of water Then

$$\mu = \frac{\sqrt{r^2 + h^2}}{r}$$

$$r = \frac{h}{\sqrt{\mu^2 - 1}}$$

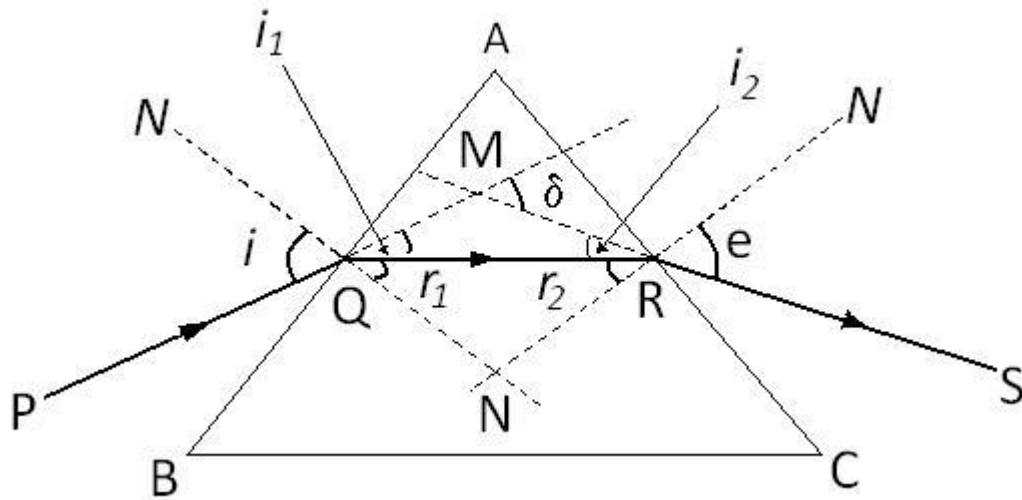
Note: (i) The radius of circular patch depends on the depth of the observer

(ii) The angular size of the cone depends only on the critical angle and is independent of the depth of the observer

(iii) For water, the vertex angle of the cone is 97.2°

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- Formula for Prism



Angle of deviation

$$\delta = (i_1 + i_2) - (r_1 - r_2)$$

Condition for angle of minimum deviation $i_1 = i_2 = i$ and $r_1 = r_2 = r$

$$\delta_m = 2i - 2r$$

$$\text{and } r = A/2$$

- Refractive index of prism material

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

For small prism (having angle $A \approx 8^\circ$ to 10°) :

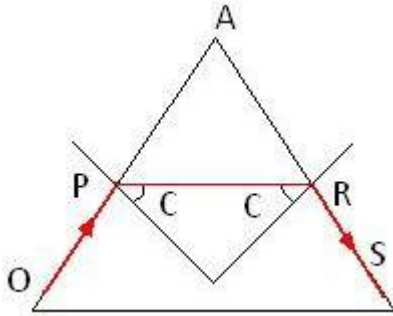
$$\delta_m = (\mu - 1) A$$

Note : since $\mu_R < \mu_V$

$$\therefore \delta_R < \delta_V$$

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- Grazing incidence and Grazing Emergence



If a light ray is incident on a prism PQR along OP , i.e. grazing along face PQ, then it emerges out of the prism in a direction RS i.e grazing along QR. Thus $i = e = 90^\circ$

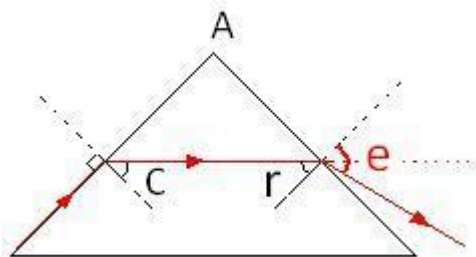
from grazing incidence and grazing emergence

$$A = 2C$$

Thus for obtaining an emergent ray from prism $A \leq 2C$

$C =$ Critical angle

- Maximum deviation by prism



$$\delta_{\text{max}} = (90 - C) + (e - r)$$

- Angle of dispersion

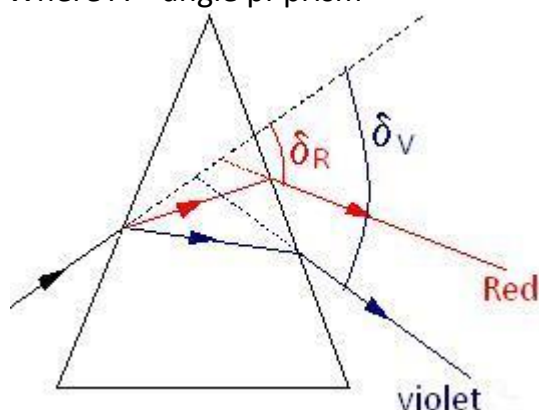
The difference in angles of deviation of any two rays is called the angle of dispersion for those rays. e,g for violet and red colours, the angle of dispersion

is

$$\theta_w = \delta_v - \delta_r = (\mu_v - \mu_r) A$$

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Where A = angle of prism



- Dispersive power

Dispersive power is the ratio of the dispersion between any two colours to the deviation suffered in the same prism by the mean of ray. the dispersive power is denoted by " ω "

$$\omega = \frac{\theta_{vr}}{\delta_y} = \left(\frac{\mu_v - \mu_r}{\mu_y - 1} \right)$$

the mean ray is the ray, the refractive index of which is the arithmetic mean of refractive index of the two colour . If μ_1 and μ_2 are the refractive indices of the two colours then the refractive index

$$\mu = \left(\frac{\mu_1 + \mu_2}{2} \right)$$

$$\omega = \frac{\mu_1 - \mu_2}{\left(\frac{\mu_1 + \mu_2}{2} - 1 \right)}$$

Note : The dispersive power " ω " is independent of the angle of the prism, therefore it does not depend on the size of the prism

Refraction at Spherical Surfaces

- Formula for refraction at convex spherical surface

If μ_1 and μ_2 are the refractive indices of first and second medium with respect

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to air then,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

power of refracting surface

$$\frac{\mu_2 - \mu_1}{\mu_2 R}$$

- Lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

- Lens makers formula

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

μ_2 is refractive index of lens material with respect to air

μ_1 is refractive index of surrounding medium with respect to air

- Newton's formula x_1 is the object distance from the focus and x_2 is image distance from second focus

f = is focal length of lens

$$f_2 = x_1 x_2$$

- Behavior of lens in medium other than air

If lens is put in a medium other than air, then three situations arises:

- If lens is immersed in a medium having refractive index less than that of lens but more than air, then the nature of the lens does not changes, only its focal length increases and power decreases
- If the lens is immersed in a medium having refractive index more than that of lens, then the nature of the lens changes i.e. convex behaves as concave and *vice-versa*. the focal length may increase, decrease or even remains same.
- If the lens is immersed in a medium having refractive index equal to that of lens then

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- (a) lens becomes invisible
- (b) The lens behaves as a plane glass plate
- (c) The focal length becomes infinite

- Power of lens

$P = 1/f$; Here f is focal length in meter

unit of power is dioptre

- Combination of lenses

When two or more lenses of focal length f_1, f_2, f_3, \dots are placed in contact with each other then focal length of combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$$

Power of lens is

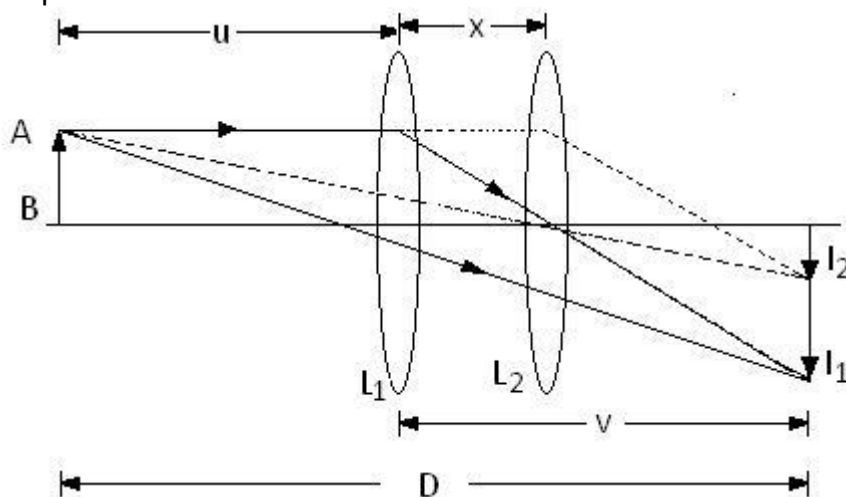
$$P = P_1 + P_2 + P_3 + \dots$$

If two thin lenses of focal length f_1 and f_2 are separated by a distance ' x ' then the effective focal length of the combination will become

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$$P = P_1 + P_2 - xP_1P_2$$

- Displacement method



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If the distance between two point pins is kept more than four times the focal length of the lens, then for two positions of the lens between the pins a real image and inverted image of one pin is formed at other. If the distance between pins is D and that between the positions of the lens is ' x ' the focal length of the lens is given by

$$f = \frac{D^2 - x^2}{4D}$$

The minimum distance between the object and its real image is $4f$

If $x > 0$; $D > 4f$. Thus distance between the two pins should be more than $4f$

If I_1 and I_2 are the heights of images in two positions of the lens, O is the size of the object then from figure

$$\frac{I_1}{I_2} = \frac{v^2}{u^2}$$

$$O = \sqrt{I_1 I_2}$$

- If convex surface of the plano-convex lens is silvered then

$$\frac{1}{f_{\text{lens}}} = +\frac{1}{(-R_{\text{lens}})} - \frac{1}{(-R_{\text{mirror}})}$$

- Lens with one surface is silvered

When one surface of thin lens is silvered, the rays are reflected back at this silvered surface. the set up acts as a spherical mirror

The focal length of the equivalent spherical mirror is given by

$$\frac{1}{f} = \sum \frac{1}{f_i}$$

Where f_i = focal length of the lens or mirror, to be repeated as many times as refraction or reflection takes place

Sign convention

In the above formula, the focal length of the converging lens or mirror positive

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and that of diverging lens or mirror negative

If one face of biconvex lens is silvered then

$$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m}$$

- Deviation produced by lens

$$\delta = h/f$$

h is the height of the light rays from principle axis

- Telescopes Astronomical telescope

Magnifying power

General formula

$$M = -\frac{f_o}{u_e}$$

f_o is focal length of objective and u_e is object distance for eye piece

(i) The final image is formed at the least distance of distinct vision

$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

Length of the telescope in this position is $L = f_o + u_e$

(ii) When final image is at infinity

$$M = -\frac{f_o}{f_e}$$

In this position the length of the telescope is maximum and is given by

$$L = f_o + f_e$$

Terrestrial Telescope

Consists of three lenses

(i) The final image is formed at the least distance of distinct vision

$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

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Length of the telescope become $L = f_o + 4f + u_e$

(ii) When final image is at infinity

$$M = -\frac{f_o}{f_e}$$

and Maximum length of telescope is $L = f_o + f_e + 4f$

Reflecting Telescope

Magnifying power

$$M = -\frac{f_o}{f_e}$$

- **Microscope**

Simple microscope

$$M = 1 + \left(\frac{D}{f}\right)$$

Compound microscope

(i) The final image is formed at the least distance of distinct vision

$$M = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e}\right)$$

Length of microscope $L = v_o + u_e$

(ii) When final image is at infinity

$$M = -\frac{v_o}{u_o} \left(\frac{D}{f_e}\right)$$

(iii) The magnifying power of reflecting telescope is 'm' = R (Radius of curvature of mirror)

- Resolving power of optical instruments (i) Resolving power of human eye = 1'

(ii) Limit of resolution of telescope

$$\Delta\theta = \frac{0.61\lambda}{a}$$

here a = radius of objective lens

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λ = wave length of radiation coming from star

(iii) Minimum separation for microscope

$$d_{min} = \frac{1.22\lambda}{2n\sin\beta}$$

- If a person can see up to a distance 'x' and wants to see the object placed at distance 'y' then the focal length of a suitable lens is

$$f = \frac{xy}{x-y}$$

- mathematical Analysis of interference

Let $Y_1 = a_1 \sin\omega t$ and $Y_2 = a_2 \sin(\omega t + \delta)$ be two simple harmonic waves of same frequency travelling in same direction, a_1 and a_2 are their amplitudes and δ is initial phase difference between them

By the principle of superposition when two or more waves reach a particle simultaneously, then resultant displacement is equal to the sum of the displacement of all waves. Hence resultant displacement is

$$Y = Y_1 + Y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

Solving we get

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$$

Resultant Intensity I given by $I = A^2$

- Conditions for maximal and minima

Maxima

The intensity I is maximum when $\cos\delta = +1$, that is when

$$\delta = 2n\pi, n = 0, 1, 2, 3, \dots$$

or path difference = $n\lambda$

$n = 0$ stands for zero order maxima

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$n=1$ stands for first maxima

$n=2$ for second maxima and so on

Thus we have $I_{\max} = (a_1 + a_2)^2$

Minima

The intensity I is minimum when $\cos\delta = -1$

That is when phase difference $\delta = (2n - 1)\pi$, where $n = 1, 2, 3, 4, \dots$

Or path difference = $(2n - 1)\lambda/2$

$n = 1$ stands for first minima,

$n = 2$ stands for second minima and so on

Thus we have $I_{\min} = (a_1 - a_2)^2$

- fringe width

$$w = D\lambda/d$$

D = distance between slit and screen, d = distance between slits

Angular width $\theta = \lambda/d$

- Position of Bright fringes

The intensity of light will be maximum at place where path difference between interfering waves is $0, \lambda, 2\lambda, \dots$ or $n\lambda$

Thus n th bright fringe we have position x_n

$$x_n = \frac{nD\lambda}{d}$$

thus by putting $n = 0, 1, 2, \dots$ we get the position of zero order, first order, second order,.... bright fringe

- Position of dark fringes

The intensity of light will be minimum at places where the path difference between the interfering waves is $\lambda/2, 3\lambda/2, 5\lambda/2, \dots$ or $(2n-1)\lambda/2$. Hence for minimum intensity we have

$$x_n = (2n - 1) \frac{D\lambda}{d}$$

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thus by $n = 1, 2, 3, \dots$ we can find out position of first, second, third ... dark fringe

- Shift of fringes by introduction of a thin lamina

When a thin sheet of any transparent material of thickness t , having refractive index μ is placed in the path of one of the two interfering rays then the fringe pattern shifts towards that ray, in the path of which the sheet is placed. the shift of the fringes is independent of the order of fringe and depends only on the wavelength, refractive index, thickness of plate and distance between the source and screen

ω = fringe width

$$\text{shift} = \frac{D}{d}(\mu - 1)t$$

$$\text{shift} = \frac{\omega}{\lambda}(\mu - 1)t$$