ELECTROSTATICS, ELECTRIC POTENTIAL, CAPACITOR

ELECTROSTATICS

ELECTRIC CHARGE

- (i) Quantization of charge : q = ±ne

  (ii) Charge on a body is independent of its velocity

  (iii) The electric charge is additive in nature.

  (iv) If a body is negatively charged then it has extra electrons, while if a body is positively charged then it has a deficit of electrons

  (v) 1 Coulomb = 3 ×10^9 stat coulomb

  (vi) The 'total' charge of a charged conductor lies at the outer surface of the conductor

  (vii) If n small drops, each of surface charge density of charge σ. Coalesce to form a big drop then the surface charge density of charge of the big drop is σ' = σ$n^{1/3}$

  (viii) If a charged conducting sphere is kept inside a hollow uncharged, insulating conducting sphere, then the total charge of the inner sphere flows to the outer sphere, on connecting the two spheres together.

  (ix) if r_1 and r_2 are the radius of two sphere having charge q_1 and q_2

  $$\frac{q_1}{q_2} = \frac{r_1}{r_2}$$

  (x) Above two spheres are connected then charge q'1 on sphere of radius r_1 and charge q'_2 of radius r_2 is

  $$q'_1 = \frac{r_1}{r_1 + r_2}(q_1 + q_2)$$

  $$q'_2 = \frac{r_2}{r_1 + r_2}(q_1 + q_2)$$

- Coulombs law

  $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where ε_o is permittivity of free space and its value is 8.86×10^{-12} coul^2/ (N×m^2)

(i) If medium is between charge is not air or vacuum then εo should be replaced by $K\varepsilon_o$ to calculate force> K is called dielectric constant of medium

(ii) Coulomb's law is true only for static charges, it does not hold good for moving charges

(iii) Coulomb force between two charges, is independent of the presence of other charges in its neighborhood.

(iv) Coulomb force between two charges acts along the line joining the two charges and depends on the distance between the charges, so it is central force

(v) Coulombs force is a vector and flows all vector laws.

(vi) If dielectric slab of thickness t is placed between the two charges separated by distance r , electric force between the charges is given by r > t

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left(r-t+\sqrt{Kt}\right)^2}$$

- Coulombs law in vector form

  if $\vec{r}_1$ and $\vec{r}_2$ are position vectors of two charges q_1 and q_2 . Force on charge q_1 due to charge q_2 is given by

$$\overrightarrow{F_{12}}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left|\vec{r}_1-\vec{r}_2\right|^3}\left(\vec{r}_1-\vec{r}_2\right)$$

- Principle of superposition

$$\vec{F}=\frac{q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i\left(\vec{r}-\vec{r}_i\right)}{\left|\vec{r}-\vec{r}_i\right|^3}$$

- equivalent distance of dielectric slab of thickness d and dielectric constant K is $\sqrt{k\,d}$
- Unit positive charge gives 4π/K lines in a medium of dielectric constant K.
- Electric field E = F/q units are N/C or V/m, dimension formula $[M\,L\,T^{-3}A^{-1}]$

- Intensity of electric field

(i) Due to Point like charge

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2}$$

for vacuum or air K = 1

(ii) Due to number of charges

The intensity at point due to number of charges q_1, q_2, q_3,... is equal to vector sum of intensities $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, ..

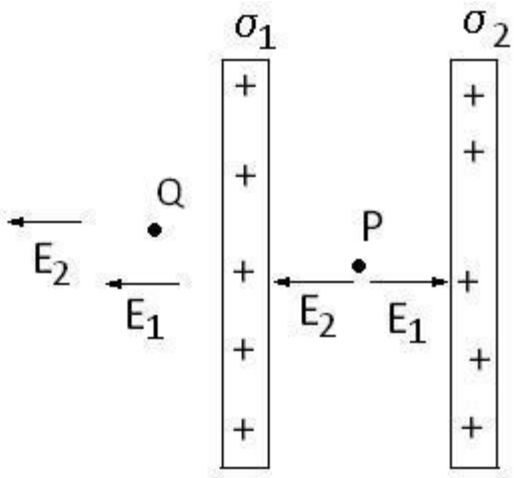
(iii) Intensity of electric field near a plane sheet of charge

$$E = \frac{\sigma}{2\varepsilon_0 K}$$

for vacuum or air K = 1

(iv) intensity of field at a point in between or outside two charged sheets

(a) Sheets charged with like charges



The intensity at point P as shown in figure

$$\vec{E}_p = E_1 - E_2 = \left(\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}\right)\hat{n}$$

If $\sigma_1 = \sigma_2$ then $E_p = 0$

The intensity at Q (as shown in figure) is

$$\vec{E}_p = E_1 + E_2 = \left(\frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}\right)\hat{n}$$

If $\sigma_1 = \sigma_2$ then

$$\vec{E}_P = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

(b) Sheets charged with opposite charges

The intensity at point P in between the plates is

$$\vec{E}_P = E_1 + E_2 = \left(\frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}\right)\hat{n}$$

(v) Intensity of electric field due to charged sphere distribution

(a) Outside the spherical charge

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2}$$

(b) On the surface of spherical charge: If R is the radius

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{R^2}$$

(c) Inside the spherical charge: E = 0

(vi) Intensity of electric field due to sphere of charge

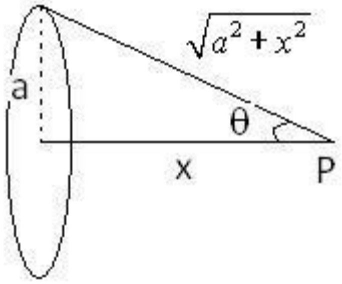
(a) on the surface and outside electric field formula is same as charged sphere

(b) inside the charged sphere

If R is the radius and r < R

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

(vii) Intensity of electric field at a point on the axis of a uniformly charged circular ring :



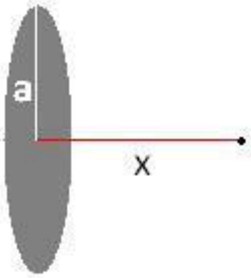
(a) Along the axis of the ring

$$E = \frac{Qx}{4\pi\varepsilon_0\left(a^2+x^2\right)^{\frac{3}{2}}}$$

(b) at the Centre of ring = 0

(c) Maximum electric field is at x = ±a/ √2

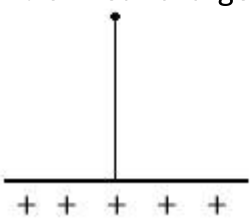
(viii) Intensity of electric field at point on the axis of a uniformly charged disc



$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\left(a^2+x^2\right)^{\frac{1}{2}}}\right]$$

(ix) Intensity of field at a point due to infinite line of charge

If λ is linear charge density



$$E = \frac{\lambda}{2\pi\varepsilon_0 y}$$

if an electron (charge e, mass m) revolves around this wire in a circular in a circular path of radius r then velocity will be

$$v = \sqrt{\frac{e\lambda}{2\pi\varepsilon_0 m}}$$

(x) Intensity of filed at a point due to dipole

length of dipole is 2l and dipole moment p = 2ql

(a) At a point on the equatorial line of dipole

$$E = \frac{p}{4\pi\varepsilon_0 K\left(r^2 + l^2\right)^{\frac{3}{2}}}$$

for air K = 1

for small dipole $r^2 >> l^2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

(b) Intensity of the electric field at point on the axis of a dipole

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

Direction of the electric field E is along the axis of the dipole from the negative charge towards positive charge

(c) General condition

If point makes an angle of θ with axis of dipole and r is the distance from the centre of dipole

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^3}\left(\sqrt{3\cos^2\theta + 1}\right)$$

(d) Electric field at axial point = 2 electric field at equator point if distance of points from the centre of dipole is same (xi) Electric field inside a uniformly charged cylinder, at a distance r form the axis is , ρ is charge density

$$E = \frac{\rho r}{2\varepsilon_0}$$

(xii) Electric field at a point closed to surface of charged conductor

$$E = \frac{\sigma}{\varepsilon_0 K}$$

- Work done by external force to rotate dipole by angle of θ from equilibrium position

  $W = pE(1-\cos\theta)$

  Thus increase in potential energy of dipole in the position θ will be given by

  $U_\theta = -pE\cos\theta$

  This is the general equation of the potential energy of the electric dipole

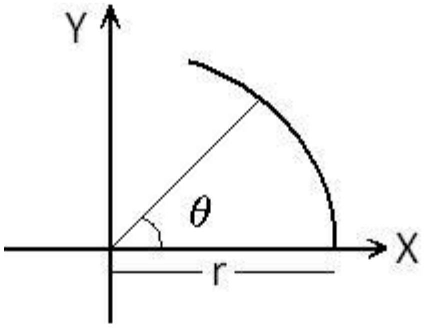
  case(i) If dipole is perpendicular to field θ = 90 then

  $U = -\ pE\cos 90 = 0$

  case(ii) If the dipole be rotated through 180° from the position of stable equilibrium, then potential energy in the new position will be

  $U = -pE\cos 180 = +pE$

  in this position the dipole will be in 'unstable equilibrium'
- A dipole placed in uniform external field experiences a torque $|\tau| = pE\sin\theta$
- Electric field at the centre due to arc of radius r and angle θ



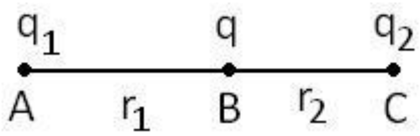
Electric field at the origin in vector form is, λ is linear charge density,

$$\vec{E} = \frac{k\lambda}{r}\left[(-\sin\theta)\hat{i} + (\cos\theta - 1)\hat{j}\right]$$

For air or vacuum K = 1

- Conditions for Equilibrium in Various Cases :

  Suppose three charges q_1, q_2 and q are situated on a straight line as shown below :



I) If q_1, q_2 are like charges and q is of unlike charge then,

(a) Force on q_1 is F_1 is

$$F_1 = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_2}{(r_1+r_2)^2} - \frac{q}{r_1^2}\right]$$

(b)Force on q_2 is F_2 is

$$F_2 = \frac{q_2}{4\pi\varepsilon_0}\left[\frac{q_1}{(r_1+r_2)^2} - \frac{q}{r_1^2}\right]$$

(c) Force on q is F

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2} - \frac{q_2}{r_2^2}\right]$$

Now, from above equations, it is clear that various equilibrium conditions can be as follows :

(i) Condition for F1 to be zero is,

$$\frac{q}{q_2} = \frac{r_1^2}{(r_1+r_2)^2}$$

(ii) Condition for F_2 to be zero

$$\frac{q}{q_1} = \frac{r_2^2}{(r_1+r_2)^2}$$

(iii) Condition for F to be zero

$$\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$$

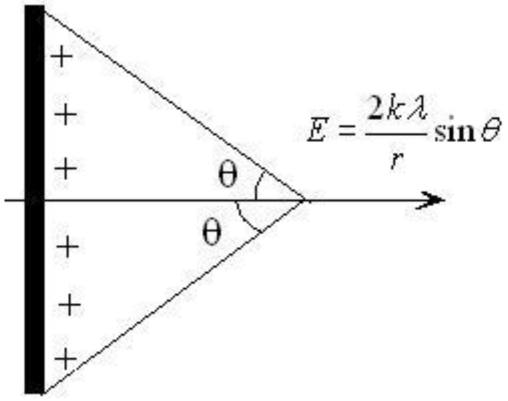
II) If q_1, q_2 and q are of same type then

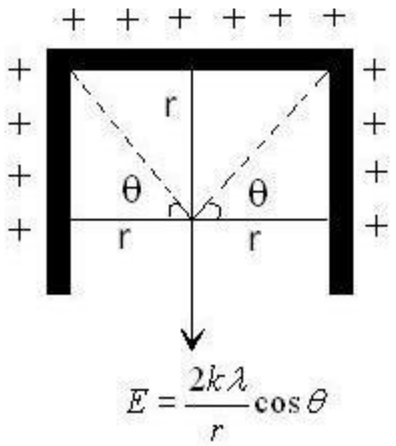
(a) Charge q will be in equilibrium, if

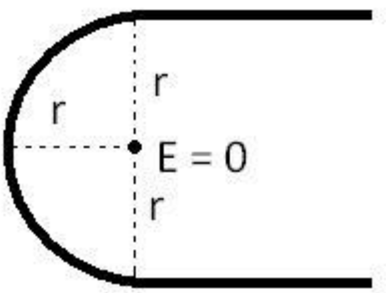
$$\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$$

(b) Charges q1 and q2 will not be in equilibrium.

- Electric field due to bending of charged rod,







ELECTRIC POTENTIAL

- Potential at point in electric field V = W/q

  Potential is scalar quantity , Units is Volts and dimensional formula is

[M L^2 T^2 $T^{-3}$$A^1$]

(i) 1 V = 1/300 stat volt

(ii) potential of earth is zero

Electric potential is uniform everywhere on the surface and inside a charged conductor

- Factors Effecting Potential of conductor

  (i) When a charged conductor is brought near another uncharged conductor, then the potential of the charged conductor is modified because of electrostatic induction

- potential difference

  Work done in moving a unit positive charge from P to Q against electric field

$$V_Q - V_P = -\int_P^Q \vec{E} \cdot \overrightarrow{dl}$$

- Potential energy U = qV

  Potential energy of system of n charges

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{kq_1q_2}{r_{ij}}$$

- Potential gradient: = dV/dl

  Relation between intensity of electric field and potential gradient

$$E = -\frac{dV}{dr}$$

If electric field have three components then

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

and

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{i} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- Equipotential surfaces: Are those surfaces at which the potential is same at all points of the surface (i) The work done in moving any charge over these surface is zero
  (ii) Equipotential surface are always normal to line of force (iii) Two equipotential surface never intersect each other (iv) Closer the equipotential surfaces, more is the intensity of electric field
- Energy stored in an Electric field (i) Energy per unit volume i.e energy density
  $U = \frac{1}{2}\varepsilon_o E^2$
  The energy associated with a spherical charge distribution of radius is
  $$U = \frac{q^2}{8\pi\varepsilon_o R}$$
  This is also the energy spent in charging the sphere
- When a surface is charged then the entire charge experiences a force at right angles to the surface.
  The outward force acting per unit area is called electrostatic pressure. Its value is
  $$P_{electrostatic} = \frac{\sigma^2}{2\varepsilon_0}$$
- When the bubble is charged then the pressure inside the bubble is reduced because the bubble inflates due to charge and its radius (or size) increases. In equilibrium condition
  (i) $\frac{4T}{a} = \frac{\sigma^2}{2\varepsilon_0}$
  (ii) The charge on bubble will be $q = 8\pi a\sqrt{(2\varepsilon 0Ta)}$
  (iii) Surface charge density of charge on the surface of the bubble is
  $$\sigma = \sqrt{\frac{8\,\varepsilon_0\,T}{a}}$$
  (iv) The intensity of the electric field on the surface of the bulb is

$$E = \sqrt{\frac{8\ T}{\varepsilon_0 a}}$$

(v) The electric potential on the surface of bubble is $V = \sqrt{\frac{8aT}{\varepsilon_0}}$

- When a metallic conductor is placed in an external electric field

  (i) A stationary charge distribution is induced on the surface of the conductor

  (ii) The resultant electric field inside the conductor is zero

  (iii) The net electric charge inside the conductor is zero

  (iv) The electric field at every point on the outer surface of conductor is locally normal to the surface

  (v) The electric potential everywhere inside the conductor is the same, constant

  (vi) If there is a cavity in the conductor then, even the conductor is placed in an external electric field, the electric field inside the conductor and also inside the cavity is always zero

- When electric charge is placed on the metallic conductor

  (i) The electric field everywhere inside the conductor is zero

  (ii) that charge is distributed only on the outer surface of the conductor

  (iii) The electric field on the surface is locally normal,

  $E = \sigma/\varepsilon_o$

  (iv) If a charge is placed inside the cavity in the conductor, the electric field in region which is outside the cavity but in the conductor remain zero. but inside the cavity it is nonzero

- When a dielectric is placed in an external electric field E_o , polarization of dielectric occurs due to electrical induction. The electric field produced by these induced charges is in the opposite direction to the direction of external electric field. Hence the resultant electric field E, inside the dielectric is less than the external electric field E_o . The dipole moment produced per unit volume is called the intensity of polarization or in short polarization

$$\vec{P}\hat{n} = \sigma_b$$

Since P ∝ E , $P = \varepsilon_o \chi_e E$, χ_e is called the electric susceptibility of the dielectric medium.

$\varepsilon_o(1+\chi_e)$ is called the permittivity ε of the dielectric medium.

$\varepsilon/\varepsilon_o$ is called the relative permittivity of that medium and it is also called the dielectric constant K.

$K = (1+\chi_e)$ $\vec{D} = (\varepsilon_o\vec{E} + \vec{P})$ is called the electric displacement. Gauss Law in the presence of dielectric is written as

$$\oint \vec{D}\cdot\vec{ds} = q$$

where q is only the net free charge

CAPACITY AND CONDENSORS

- Capacity of a conductor : C = Q/V

  Units of capacitance Farad, 1 microfarad(1μF)= 10^{-6}Farad,

  1 picofarad(1pF) = 10^{-12}Farad

  1 farad = 1 coul/volt

  F Farad = 9×10^{11} stat Farad

  Dimensions of capacitance : $[M^{-1} L^{-2} T^4 A^2]$

- Capacitance formula

  (i) Spherical conductor of radius r , $C = 4\pi\varepsilon_o K r$, for air K = 1

  (ii) parallel plates capacitor , A is area of plate , d is distance between the plates

  $$C = \frac{K\varepsilon_0 A}{d}$$

  For air K = 1

  If C_o is the capacitance when there is air (vacuum) in plates then

  $C = KC_o$

  (iii) Capacitance of a parallel plate capacitor partly filled with dielectric of thickness t

$$C = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$$

case(a) if slab of metal of thickness between the plates, then capacitance will be

$$C = \frac{\varepsilon_0 A}{(d - t)}$$

case(b) If several slabs of dielectric constants K_1, K_2, K_3,.. and having thickness t_1, t_2, t_3, ... be placed between the plates then capacitance

$$C = \frac{\varepsilon_0 A}{\left(d - (t_1 + t_2 + ..) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots\right)\right)}$$

If $d = t_1 + t_2 + t_3 + ...$ then

$$C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots\right)}$$

(iv) Spherical capacitor, a is radius of inner sphere and b is radius of outer sphere Outer sphere is earthed

$$C = 4\pi\varepsilon_0 K\left(\frac{ab}{b - a}\right)$$

case: If inner sphere is earthed and outer sphere is charged then capacitance

$$C = 4\pi\varepsilon_0 Kb + 4\pi\varepsilon_0 K\left(\frac{ab}{b - a}\right)$$

(v) Cylindrical condenser, r_1 is radius of inner cylinder and r_2 is the radius of outer cylinder , l is the length of cylinder

$$C = \frac{2\pi\varepsilon_0 lK}{2.303\log_{10}\left(\frac{r_2}{r_1}\right)}$$

- Potential energy of a charged conductor/ capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$$

- Combination of capacitor

(i) Capacitor connected in parallel

$C = C_1 + C_2 + C_3 + \ldots$

(ii) Capacitance connected in Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

(iii) Common potential when two capacitors are connected

Potential on C_1 be V_1 and potential of C_2 be V_2 then Common potential V after connecting is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(i) If charges before connections are q1 and q 2 respectively then after connecting charges be q'_1 and q'_2 then $q'_1 = C_1V$ and $q'_2 = C_2V$

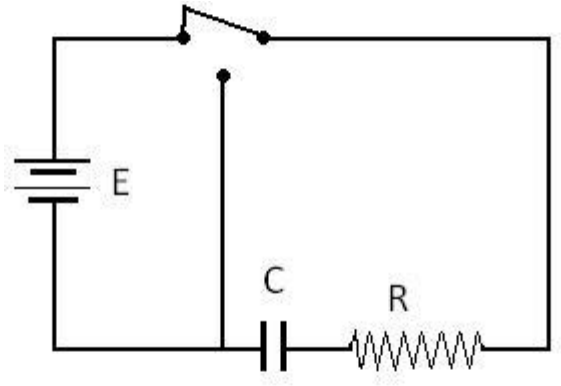
or

$$\frac{q'_1}{q'_2} = \frac{C_1}{C_2}$$

(ii) Loss of energy in redistribution of charges

$$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

- Charging of a condenser/ capacitor through a resistance



emf of batter E, q_o is charge on capacitor when it is fully charged, Io is steady state current at time t = ∞

V is potential across capacitance, q is charge of capacitor at time t

$$q = q_0 \left(1 - e^{\frac{-t}{CR}}\right)$$

$$V = V_0 \left(1 - e^{\frac{-t}{CR}}\right)$$

$$I = I_0 \left(e^{\frac{-t}{CR}}\right)$$

RC is called time constant of the C-R circuit when t = CR then

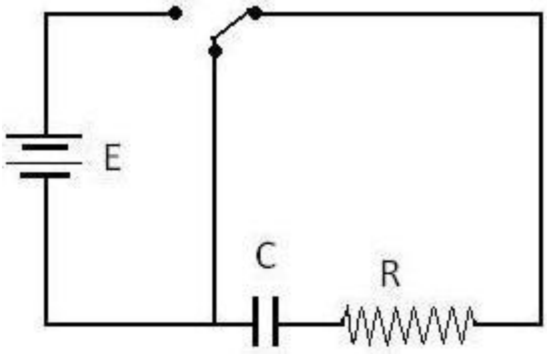
q = $0.632I_o$

V = 0.632E

I = - $0.37I_o$

Thus time constant is that time each the charging current decays to 0.37 of maximum current or the time in which voltage grows to 0.632 times of ,maximum charging voltage

Discharging of a condenser through a resistance



E is initial potential across capacitor , q o is the initial charge , Io is initial discharging current

V is potential across capacitor at time t , I is the current at time t , q is the charge at time t

Then

$$q = q_0\, e^{\frac{-t}{CR}}$$

$$V = V_0\ e^{\frac{-t}{CR}}$$

$$I = I_0\ \left(e^{\frac{-t}{CR}}\right)$$

RC is time constant is that time in which the charge decays to 0.37 of the maximum value

- If n small drop each having a charge q and capacity C and potential V, and surface charge density σ coalesce to form a big drop the
  (i) The charge on the big drop = nq
  (ii) Capacity of big drop = $n^{1/3}$ C
  (iii) Potential of big drop = $n^{2/3}$ V
  (iv) Potential energy of big drop = $n^{5/3}$ U
  (v) surface charge density σ' = $n^{1/3}$ σ
- If parallel plate capacitor is charged to V volts and the charging battery is removed.

A dielectric plate is now introduced between the plates of the condenser, then

(i) Charge on the condenser remains the same

(ii) Capacity increases, becomes K times the previous value

(iii) Potential difference between the plates decreases and becomes 1/K times the previous value

(iv) The field between the plates decreases and becomes 1/K times the previous value

(v) Energy decreases and becomes 1/K times the previous value i.e. U' =U/K

- If the air capacitor is charged to V volts and a dielectric plate is introduced in the gap between the plates , battery is till connected, then

(i) The charging on the plates increases

(ii) The capacity increases K times

(iii) The potential difference between the plates remains the same

(iv) Field between the plates remains the same

(v) The energy increases, becomes K times the previous value U' = KU